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Another strongly exceptional collection of coherent sheaves on a Grassmannian[☆]



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ABSTRACT

On the Grassmannian of 2-dimensional subspaces in a finite dimensional linear space we construct a Karoubian complete strongly exceptional PO set of coherent sheaves, parametrized by the cosets of the Weyl group of the general linear group of the linear space modulo the Weyl group of the parabolic subgroup stabilizing the subspace, from subquotients of the Frobenius direct image of the structure sheaf of the Grassmannian defined over a field of large positive characteristic. Our collection diverges from the one discovered by Kapranov. We also show in the general setting over any field of positive characteristic that the sheaf corresponding to the longest element of the cosets is a direct summand of the Frobenius direct image of the structure sheaf of the homogeneous space.

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Introduction

Let $\mathcal{P}$ be a smooth homogeneous projective variety over an algebraically closed field $\mathbb{k}$ of positive characteristic p . Write $\mathcal{P} = G/P$ with a simply connected semisimple algebraic

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group G over $\mathbb{k}$ and P a parabolic subgroup of G . In [13] on the projective spaces and in [15] on the quadrics we have constructed for large p a Karoubian complete strongly exceptional PO set of coherent modules on $\mathcal{P}$, parametrized by the cosets of the Weyl group of G by the Weyl group of P , from certain subquotients of the Frobenius direct image of the structure sheaf of $\mathcal{P}$. Those sheaves are all defined over $\mathbb{Z}$, and give a collection over a field of characteristic 0 by base change. On the projective spaces the collection consists of invertible sheaves, coinciding with the one discovered by Beilinson [3] while the one on the quadrics is slightly different in type D from the one found by Kapranov [18]. In this paper we show that the same recipe yields a Karoubian complete strongly exceptional PO set on the Grassmannian with P stabilizing a 2 dimensional subspace, which diverges further from the one discovered by Kapranov [17]. The observation that such a construction might work goes back to [16], which dealt with G of rank 2. To put it in perspective of geometric representation theory/algebraic geometry, see [4,22,6,20,8].

To describe our program, let B be a Borel subgroup of G , T a maximal torus of B , Λ the character group of T , R the root system of G relative to T , R^+ the positive system of R such that the roots of B are $-R^+$, R^s the set of simple roots of R^+ , W the Weyl group of G . Put $\rho = \frac{1}{2} \sum_{\alpha \in R^+} \alpha$. For each simple root α let $\varpi_\alpha \in \Lambda$ denote the corresponding fundamental weight. Besides the standard action of W on Λ we let W act on Λ by shifting the origin to $-\rho$; $\forall w \in W$, $\lambda \in \Lambda$, we write $w \bullet \lambda = w(\lambda + \rho) - \rho$. Let $I \subset R^s$ be the set of the simple roots of the standard Levi subgroup L_P of P , and let $W^P = \{w \in W | w\alpha > 0 \ \forall \alpha \in I\}$. If W_I is the Weyl group of L_P , $W = \sqcup_{w \in W^P} wW_I$. For each $w \in W^P$ let $\mu_w = w^{-1} \bullet (-\sum_{\substack{\alpha \in R^s \\ w^{-1}\alpha < 0}} \varpi_\alpha)$. Let $\nabla^P = \text{ind}_B^P$ denote

the induction functor from the category of B -modules to the category of P -modules [11, I.3], and let $\mathcal{E}(w) = \mathcal{L}_{\mathcal{P}}(\nabla^P(\mu_w))$ be the locally free sheaf on $\mathcal{P}$ associated to the P -module $\nabla^P(\mu_w)$ of highest weight μ_w . We showed in [13] (resp. [15]) that if $\mathcal{P}$ is a projective space (resp. quadric with p odd), the $\mathcal{E}(w)$, $w \in W^P$, form a Karoubian complete strongly exceptional PO set such that $\text{Mod}_{\mathcal{P}}(\mathcal{E}(x), \mathcal{E}(y)) \neq 0$ iff $x \geq y$ in the Chevalley–Bruhat order. In general, more complex P -modules than $\nabla^P(\mu_w)$ enter [16]. Letting G_1 denote the Frobenius kernel of G , the Frobenius direct image $F_*\mathcal{O}_{\mathcal{P}}$ of the structure sheaf $\mathcal{O}_{\mathcal{P}}$ of $\mathcal{P}$ may be identified with the sheaf $\mathcal{L}_{G/G_1P}(\hat{\nabla}^P(\mathbb{k}))$ over G/G_1P associated to the G_1P -Verma module $\hat{\nabla}_P(\mathbb{k}) = \text{ind}_{P_1}^{G_1P}(\mathbb{k})$ induced from the trivial 1-dimensional P -module $\mathbb{k}$. If $\Lambda_1 = \{\lambda \in \Lambda \mid \langle \lambda, \alpha^\vee \rangle \in [0, p] \ \forall \alpha \in R^s\}$ the set of p -restricted dominant weights, one can write any $\lambda \in \Lambda$ as $\lambda = \lambda^0 + p\lambda^1$ with $\lambda \in \Lambda_1$ and $\lambda^1 \in \Lambda$. Let $L(\lambda^0)$ be the simple G -module of highest weight λ^0 , which remains simple as G_1 -module. If p is large enough that Lusztig’s conjecture on the irreducible characters of G holds [11, C], each $L((w \bullet 0)^0) \otimes_{\mathbb{k}} \nabla^P(w^{-1} \bullet (w \bullet 0)^1)^{[1]}$, $w \in W^P$, appears as a subquotient of $\hat{\nabla}_P(\mathbb{k})$ [1,15], where $M^{[1]}$ denotes the Frobenius twist of a P -module M . Thus, $\mathcal{L}_{G/G_1P}(\nabla^P(w^{-1} \bullet (w \bullet 0)^1)^{[1]}) \simeq \mathcal{L}_{\mathcal{P}}(\nabla^P(w^{-1} \bullet (w \bullet 0)^1))$ is a subquotient of $F_*\mathcal{O}_{\mathcal{P}}$, and our μ_w is equal to $w^{-1} \bullet (w \bullet 0)^1$.

Now, let E be an n -dimensional $\mathbb{k}$ -linear space of basis e_i , $i \in [1, n]$, $G = \text{GL}(E)$, B the Borel subgroup of G consisting of lower triangular matrices, T the maximal torus

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