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Mac Lane (co)homology of the second kind and Wieferich primes

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ABSTRACT

In this paper we investigate the connection between the Mac Lane (co)homology and Wieferich primes in finite localizations of global number rings. Following the ideas of Polishchuk–Positselski [29], we define the Mac Lane (co)homology of the second kind of an associative ring with a central element. We compute these invariants for finite localizations of global number rings with an element w and obtain that the result is closely related to the Wieferich primes to the base w. In particular, for a given non-zero integer w, the infiniteness of Wieferich primes to the base w turns out to be equivalent to the following: for any positive integer n, we have $HML^{II,0}(\mathbb{Z}[\frac{1}{n!}], w) \neq \mathbb{Q}$.

As an application of our technique, we identify the ring structure on the Mac Lane cohomology of a global number ring and compute the Adams operations (introduced in this case by McCarthy [26]) on its Mac Lane homology.

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Contents

1. Introduction

In this paper we investigate the connection between the Mac Lane (co)homology and Wieferich primes in finite localizations of global number rings.

We first recall the motivating geometric picture. Let $X = \operatorname{Spec} B$ be a smooth affine variety over a field k of characteristic zero. By Hochschild–Kostant–Rosenberg theorem [15], we have isomorphisms

$$HH_n(B) \cong \Omega^n(X), \quad HH^n(B) \cong \Lambda^n T_X, \quad n \geq 0,$$

where $HH_\bullet$ and $HH^\bullet$ denote the Hochschild homology and cohomology, which are recalled in Section 7.1.

In the paper [29], Polishchuk and Positselski define and thoroughly study the notion of Hochschild homology $HH^{II}_\bullet(\mathcal{C}, M)$ (resp. cohomology $HH^{II,\bullet}(\mathcal{C}, M)$) of the second kind of a curved DG category $\mathcal{C}$ with coefficients in a curved DG bimodule M. As usual, one writes simply $HH^{II}_\bullet(\mathcal{C})$ (resp. $HH^{II,\bullet}(\mathcal{C})$) when M is the diagonal bimodule.

Within the above notation, any element $w \in B$ gives a $\mathbb{Z}/2$-graded curved DG algebra B_w, which equals B (concentrated in degree zero) as a $\mathbb{Z}/2$-graded algebra, has zero differential and curvature w. It was shown in [7] that we have natural isomorphisms

$$HH^{II}_0(B_w) \cong \bigoplus_i H^{2i}(\Omega^\bullet(X), dw\wedge), \quad HH^{II}_1(B_w) \cong \bigoplus_i H^{2i-1}(\Omega^\bullet(X), dw\wedge).$$