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Auslander–Reiten quiver of type D and generalized quantum affine Schur–Weyl duality

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ABSTRACT

We first provide an explicit combinatorial description of the Auslander–Reiten quiver Γ_Q of finite type D. Then we can investigate the categories of finite dimensional representations over the quantum affine algebra $U_q'(D^{(i)}_{n+1})$ $(i = 1, 2)$ and the quiver Hecke algebra $R_{D_{n+1}}$ associated to D_{n+1} $(n \geq 3)$, by using the combinatorial description and the generalized quantum affine Schur–Weyl duality functor. As applications, we can prove that Dorey's rule holds for the category $\mathrm{Rep}(R_{D_{n+1}})$ and prove an interesting difference between multiplicity free positive roots and multiplicity non-free positive roots.

1. Introduction

The quiver Hecke algebras, introduced by Khovanov–Lauda [25,26] and Rouquier [33], are in the limelight among the people in the representation research area because the algebras are related to categorification of quantum groups. Recently, the quiver Hecke algebras attract the people's attention once again because the algebras can be understood

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as a generalization of the affine Hecke algebra of type A in the context of the quantum affine Schur–Weyl duality, which makes bridge between the representations of quiver Hecke algebras and the quantum affine algebras $U_q'(\mathfrak{g})$, by the results of Kang, Kashiwara and Kim [21,22].

For the quantum affine algebra $U_q'(\mathfrak{g})$, the finite dimensional integrable representations over $U_q'(\mathfrak{g})$ have been investigated by many authors during the past twenty years from different perspectives (see [1,7,8,12,15,23,28]). Among these aspects, the theory of R-matrix provides crucial information for constructing the quantum affine Schur–Weyl duality functor in [21,22] (see also [10,11,14,19]).

As a continuation of the previous series paper [30], we provide an explicit combinatorial description of the Auslander–Reiten (AR) quiver Γ_Q of finite type D and apply the combinatorial description to investigate

(i) the category $\mathscr{C}_Q^{(i)}$ $(i = 1, 2)$, consisting of finite dimensional integrable modules over the quantum affine algebra $U_q'(D_{n+1}^{(i)})$ depending on the AR-quiver Γ_Q ([16,18]),
(ii) the category $\mathrm{Rep}(R_{D_{n+1}})$, consisting of finite dimensional graded modules over the quiver Hecke algebra $R_{D_{n+1}}$ associated to D_{n+1} $(n \geq 3)$,

with the exact quantum affine Schur–Weyl duality functor

$$\mathcal{F}_Q^{(1)} : R_{D_{n+1}} \longrightarrow \mathscr{C}_Q^{(1)}.$$

Here Q is any Dynkin quiver of finite type D_{n+1} by orienting edges of Dynkin diagram of finite type D_{n+1}.

Let Φ_n^+ be the set of all positive roots associated to finite Dynkin diagram of finite type A_m, D_m, E_6, E_7 or E_8. Then it is well-known ([13]) that

(i) the vertices of Γ_Q can be identified with the set Φ_m^+ and the set of all isomorphism classes of indecomposable modules over the path algebra $\mathbb{C}Q$,
(ii) the dimension vector of indecomposable module corresponding to $\beta \in \Gamma_Q$ is indeed β,
(iii) arrows in Γ_Q present the irreducible morphisms between the indecomposables,
(iv) Γ_Q provides *the unique convex partial order* $\prec_Q$ on Φ_m^+ which is compatible with paths in Γ_Q ([4]).

Note that each positive root β in Φ_n^+ of finite type D_n can be expressed by a pair of integers $\{a, \pm b\}$ $(1 \leq a < b \leq n)$ where $\beta = \varepsilon_a \pm \varepsilon_b$. We say ε_a and $\pm\varepsilon_b$ are summands of β. Identifying β with $\{a, \pm b\}$, every positive root appearing in the *maximal N-sectional* (resp. *S-sectional*) path and *the maximal swing* in Γ_Q share the same summand as ε_a or $\pm\varepsilon_b$ (Theorem 2.21, Theorem 2.25).

With the explicit combinatorial description of Γ_Q of finite type D_n, we can prove that the *Dorey's rule* in [9] always holds for all $\alpha \prec_Q \beta \in \Phi_n^+$ with $\gamma = \alpha + \beta \in \Phi_n^+$ (Section 4); i.e., the following surjective homomorphisms exist (see Definition 2.3 for ϕ^{-1}):