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Automorphisms of restricted parabolic trees and Sylow p-subgroups of the finitary symmetric group

Agnieszka Bier [a,*], Yuriy Leshchenko [b], Vitaliy Sushchanskyy [a]

[a] *Institute of Mathematics, Silesian University of Technology, ul. Kaszubska 23, 44-100 Gliwice, Poland*
[b] *Institute of Physics, Mathematics and Computer Science, Cherkasy National University, Shevchenko blvd. 79, Cherkasy 18031, Ukraine*

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In the paper we introduce the notion of a k-adic restricted parabolic tree D_k and investigate the group $Aut\ D_k$ of automorphisms of this tree. In particular, we characterize the Sylow p-subgroups in the subgroup $Aut_f\ D_p$ of finitary automorphisms of a p-adic restricted parabolic tree. Then we use the characterization for the classification of Sylow p-subgroups in the finitary symmetric group $FS_{\mathbb{N}}$.

* Corresponding author.
E-mail addresses: agnieszka.bier@polsl.pl (A. Bier), ylesch@ua.fm (Y. Leshchenko), vitaliy.sushchanskyy@polsl.pl (V. Sushchanskyy).

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1. Introduction

In the past few decades the groups acting on trees have found many applications, especially in group theory, harmonic analysis, geometry, dynamics and representation theory. For instance, in group theory the groups acting on trees provide constructions of groups with specific properties. These constructions usually employ the group of automorphisms of an infinite homogeneous rooted tree.

Let (T, v_0) denote the rooted tree with root v_0. The set $V(T)$ of all vertices of T is partitioned into subsets of vertices lying at the same distance to the root v_0. The set L_i of vertices on distance i to the root is called the *i-th level* of the tree. The tree is called *spherically homogeneous*, if for every i there exists a number k_i such that for every vertex $v \in L_i$ the number of elements of L_{i+1} which are adjacent to v is equal to k_i. In this case the vector $\overline{k}_n = (k_1, k_2, \dots, k_{n-1})$ is called the *spherical index* of T, and the tree is denoted by $(T_{\overline{k}_n}, v_0)$. If $\overline{k}_n = (k, k, \dots, k)$, then $(T_{\overline{k}_n}, v_0)$ is called k-adic and denoted by $(T_{k,n}, v_0)$.

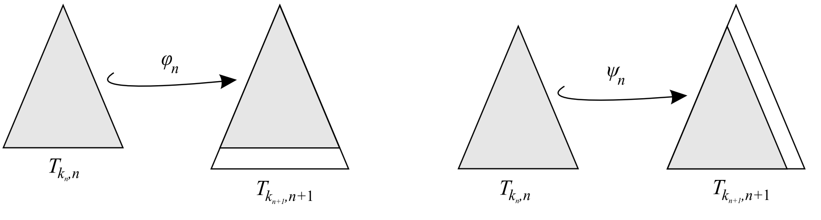
The infinite homogeneous rooted tree $(T_{\overline{k}}, v_0)$ with root v_0 and spherical index $\overline{k} = (k_1, k_2, \dots)$ is the direct limit

$$(T_{\overline{k}}, v_0) \cong \varinjlim_{n} \left((T_{\overline{k}_n}, v_0), \varphi_n\right),$$

of finite homogeneous rooted trees $(T_{\overline{k}_n}, v_0)$ with root v_0, spherical index $\overline{k}_n = (k_1, k_2, \dots, k_{n-1})$ and embeddings $\varphi_n : (T_{\overline{k}_n}, v_0) \hookrightarrow (T_{\overline{k}_{n+1}}, v_0)$ shown in Fig. 1a.

The group $Aut\,(T_k, v_0)$ of automorphisms of an infinite k-adic ($k \geq 2$) rooted tree (T_k, v_0) is an object of particular interest and has been widely investigated. For instance, it contains subgroups which are just infinite groups, groups of intermediate growth or Burnside type groups. A lot of interesting results have been obtained in this direction by L. Bartholdi, R. Grigorchuk, S. Sidki, V. Nekrashevych and others (see e.g. [2,18]). Moreover, certain groups acting on infinite rooted trees initialized the studies of self-similar group actions on spaces [3,15]. Another interesting result concerns the distribution of orders of random elements and their Hausdorff dimension of automorphism groups of the infinite homogeneous rooted tree [1].

The group $Aut\,(T_k, v_0)$ is profinite and hence the Sylow theorems are valid: for every prime p there exists a Sylow p-subgroup of $Aut\,(T_k, v_0)$ (in a topological sense) and



(a) Embedding φ_n (b) Embedding ψ_n

Fig. 1. Embeddings of homogeneous rooted trees.