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Morita equivalences and Azumaya loci from Higgsing dimer algebras

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A B S T R A C T

Let $\psi : A \to A'$ be a cyclic contraction of dimer algebras, with A non-cancellative and A' cancellative. A' is then prime, noetherian, and a finitely generated module over its center. In contrast, A is often not prime, nonnoetherian, and an infinitely generated module over its center. We present certain Morita equivalences that relate the representation theory of A with that of A'.

We then characterize the Azumaya locus of A in terms of the Azumaya locus of A', and give an explicit classification of the simple A-modules parameterized by the Azumaya locus. Furthermore, we show that if the smooth and Azumaya loci of A' coincide, then the smooth and Azumaya loci of A coincide. This provides the first known class of algebras that are nonnoetherian and infinitely generated modules over their centers, with the property that their smooth and Azumaya loci coincide.

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1. Introduction

1.1. Main results

The main objective of this paper is to characterize Morita equivalences and Azumaya loci of non-cancellative dimer algebras. Dimer algebras were introduced in string theory, and have wide application to many areas of mathematics, such as noncommutative crepant resolutions, Calabi–Yau algebras, the McKay correspondence, mirror symmetry, wall-crossing, and cluster algebras (e.g., [6,7,15,9,14,11,12,10,1]).

A dimer algebra is a quiver algebra whose quiver embeds into a two-torus, with homotopy-like relations derived from a potential (precise definitions will be given in Section 1.2). Cancellative dimer algebras are Calabi–Yau algebras and noncommutative crepant resolutions ([5, Theorem 10.2], [7], [9, Theorem 4.3], [15, Theorem 6.3]). In contrast, non-cancellative dimer algebras are nonnoetherian, infinitely generated modules over their centers, and often not prime or PI [4, Theorems 1.3, 4.17, 4.47]. However, almost all dimer algebras are non-cancellative. Furthermore, non-cancellative dimer algebras correspond to certain unstable quiver gauge theories which may describe the vacuum moduli space at sufficiently high energies. It is therefore of great interest to understand these highly complex algebras, both from a mathematical and a string theoretic perspective.

Our primary tool in studying non-cancellative dimer algebras is a 'cyclic contraction', introduced in [4]. Roughly, a cyclic contraction is a k-linear map of dimer algebras $\psi : A = kQ/I \to A' = kQ'/I'$, where Q' is obtained by contracting a set of arrows of Q to vertices, such that (i) A' is cancellative, and (ii) the commutative algebras generated by the cycles of Q and Q' coincide. This common algebra, denoted S, is isomorphic to the center of A' and is called the 'cycle algebra' of A. An example of a cyclic contraction is given in Fig. 1.

Recall that two rings are Morita equivalent if they have equivalent module categories. Our main results are the following.

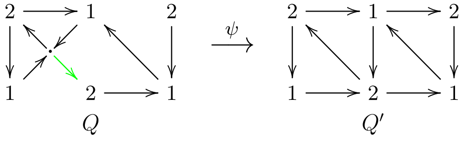

Fig. 1. The non-cancellative dimer algebra $A = kQ/I$ cyclically contracts to the cancellative dimer algebra $A' = kQ'/I'$. Both quivers are drawn on a torus, and the contracted arrow is drawn in green. The cycle algebra of A is $S = k\left[x^2, y^2, xy, z\right] \subset B = k[x, y, z]$, and the reduced center of A is $\hat{Z} = k + (x^2, y^2, xy)S$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)