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Unphysical diagonal modular invariants

Alexei Davydov

Department of Mathematics, Ohio University, Athens, OH 45701, USA

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ABSTRACT

A modular invariant for a chiral conformal field theory is physical if there is a full conformal field theory with the given chiral halves realising the modular invariant. The easiest modular invariants are the charge conjugation and the diagonal modular invariants. While the charge conjugation modular invariant is always physical there are examples of chiral CFTs for which the diagonal modular invariant is not physical. Here we give (in group theoretical terms) a necessary and sufficient condition for diagonal modular invariants of G-orbifolds of holomorphic conformal field theories to be physical.

Mathematically a physical modular invariant is an invariant of a Lagrangian algebra in the product of (chiral) modular categories. The chiral modular category of a G-orbifold of a holomorphic conformal field theory is the so-called (twisted) Drinfeld centre $\mathcal{Z}(G,\alpha)$ of the finite group G. We show that the diagonal modular invariant for $\mathcal{Z}(G)$ is physical if and only if the group G has a *double class inverting* automorphism, that is an automorphism $\phi : G \to G$ with the property that for any commuting $x, y \in G$ there is $g \in G$ such that $\phi(x) = gx^{-1}g^{-1}$, $\phi(y) = gy^{-1}g^{-1}$.

Groups without double class inverting automorphisms are abundant and provide examples of chiral conformal field theories for which the diagonal modular invariant is unphysical.

E-mail address: alexei1davydov@gmail.com.

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1. Introduction

The aim of this note is to construct examples of chiral rational conformal field theories with the same chiral algebras for which the diagonal modular invariant is not physical.

Recall the state space of a (2-dimensional) conformal field theory comes equipped with amplitudes associated to a finite collection of fields inserted in marked points of a Riemann surface [11,23]. Fields, whose amplitudes depend (anti-)holomorphically on insertion points, form what is known as (anti-)chiral algebra of the CFT. The state space is naturally a representation of the product of the chiral and anti-chiral algebras. A conformal field theory is *rational* if the state space is a finite sum of tensor products of irreducible representations of chiral and anti-chiral algebras. The matrix of multiplicities of irreducible representations in the decomposition of the state space is called the *modular invariant* of the RCFT. The simplest case (the so-called *Cardy case*) is the case of the *charge conjugation* modular invariant, which assumes that chiral and anti-chiral algebras coincide. Another very natural case is the *diagonal* modular invariant.

The name modular invariant comes from the fact that the matrix of multiplicities is invariant with respect to the modular group action on characters. This fact was used in [3] to classify modular invariants for affine $sl(2)$ rational conformal field theories. This paper started an activity aimed at classifying possible modular invariants for various conformal field theories. It took some time to realise that not all modular invariants correspond to conformal field theories, in other words there are *unphysical* modular invariants [15,27]. In the paper [7] examples of different rational conformal field theories with the same (charge conjugation) modular invariant were constructed. Thus although being a convenient numerical invariants of a rational conformal field theory modular invariants are far from being complete.

Mathematical descriptions of rational conformal field theories were obtained relatively recently (see [14] and the references therein). One of the mathematical axiomatisations