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The period-index problem of the canonical gerbe of symplectic and orthogonal bundles

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ABSTRACT

We consider regularly stable parabolic symplectic and orthogonal bundles over an irreducible smooth projective curve over an algebraically closed field of characteristic zero. The morphism from the moduli stack of such bundles to its coarse moduli space is a μ_2-gerbe. We study the period and index of this gerbe, and solve the corresponding period-index problem.

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1. Introduction

Let X be a smooth, projective, irreducible curve over an algebraically closed field k of characteristic zero. A symplectic (respectively, orthogonal) vector bundle with parabolic structure on X consists of a parabolic vector bundle (E_*, B) with a skew-symmetric (respectively, symmetric) pairing

$$E_* \otimes E_* \longrightarrow \mathcal{O}_X,$$

satisfying a nondegeneracy condition (see Section 2). Note that the above tensor product is a tensor product in the category of parabolic vector bundles.

A regularly stable symplectic (respectively, orthogonal) parabolic bundle is one whose automorphism group coincides with the center of the symplectic (respectively, orthogonal) group. Let $\mathrm{Bun}_G^{\mathrm{rs}}$ be the moduli stack of regularly stable symplectic or orthogonal parabolic bundles, and let M_G^{rs} be the corresponding coarse moduli space. We have a μ_2-gerbe

$$\mathrm{Bun}_G^{\mathrm{rs}} \longrightarrow M_G^{\mathrm{rs}}.$$

The purpose of this paper is to study the period-index problem for this gerbe. The center of $\mathrm{SO}(2n+1)$ is trivial, hence in this case $\mathrm{Bun}_G^{\mathrm{rs}} = M_G^{\mathrm{rs}}$. Therefore, we will assume that the rank in the orthogonal case is even.

The index is computed in Theorem 7.4. The main idea is to degenerate the gerbe to a highly singular point (see Proposition 2.4) and to use Luna's étale slice theorem to study the geometry of this stack over the moduli space here.

In Section 2, we briefly recall the definition and properties of parabolic bundles on X, and recall the construction of their moduli spaces. In Proposition 2.2 we compute the period of the canonical gerbe in most cases. We also discuss an application of Luna's étale slice theorem to our case using Proposition 2.4. In Section 3, after a brief overview of twisted sheaves, we give an upper bound for the index of the canonical gerbe.

In Sections 4 and 5, we discuss the concept of a stable central simple algebra with involution, and prove that for a stable central simple algebra with involution over a field F, there exists a morphism $\mathrm{Spec}\, F \longrightarrow Z^s/G^{ad}$, where G denotes an appropriate symplectic or orthogonal group. Finally, in Section 6, we prove the existence of lower bounds for the index of the canonical gerbe. This gives the index completely in the symplectic case, and it gives a very strong lower bound for the orthogonal case. Appendices A and B discuss a technical result used in the proof, and symplectic or orthogonal involutions on a central simple algebra.

1.1. Conventions

- We work over an algebraically closed base field k of characteristic zero.
- By G we denote $\mathrm{Sp}(2n)$ or $\mathrm{SO}(2n)$, and G^{ad} denotes the adjoint form of G, i.e. $\mathrm{PSp}(2n)$ or $\mathrm{PSO}(2n)$. In the case of $G = \mathrm{SO}(2n)$, we assume that $n \geq 2$. Also, $G(j)$