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Palindromic automorphisms of free groups

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ABSTRACT

Let F_n be the free group of rank n with free basis $X = \{x_1, \dots, x_n\}$. A palindrome is a word in $X^{\pm 1}$ that reads the same backwards as forwards. The palindromic automorphism group ΠA_n of F_n consists of those automorphisms that map each x_i to a palindrome. In this paper, we investigate linear representations of ΠA_n, and prove that ΠA_2 is linear. We obtain conjugacy classes of involutions in ΠA_2, and investigate residual nilpotency of ΠA_n and some of its subgroups. Let IA_n be the group of those automorphisms of F_n that act trivially on the abelianisation, PI_n be the palindromic Torelli group of F_n, and let $E\Pi A_n$ be the elementary palindromic automorphism group of F_n. We prove that $PI_n = IA_n \cap E\Pi A'_n$. This result strengthens a recent result of Fullarton [2].

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1. Introduction

Let F_n be the free group of rank n with free basis $X = \{x_1, \ldots, x_n\}$, and let $Aut(F_n)$ be the automorphism group of F_n. A reduced word $w = x_1^{\epsilon_1} \ldots x_n^{\epsilon_n}$ in $X^{\pm 1}$ is called a palindrome if it is equal to its reverse word $\overline{w} = x_n^{\epsilon_n} \ldots x_1^{\epsilon_1}$. In [1], Collins defined the palindromic automorphism group ΠA_n as the subgroup of $Aut(F_n)$ consisting of those automorphisms that map each x_i to a palindrome. He proved that ΠA_n is finitely presented, and that it is generated by the following three types of automorphisms:

$$t_i : \begin{cases} x_i \longmapsto x_i^{-1} \\ x_k \longmapsto x_k \quad \text{for } k \neq i, \end{cases}$$

$$\alpha_{i,i+1} : \begin{cases} x_i \longmapsto x_{i+1} \\ x_{i+1} \longmapsto x_i \\ x_k \longmapsto x_k \quad \text{for } k \neq i, \end{cases}$$

$$\mu_{ij} : \begin{cases} x_i \longmapsto x_j x_i x_j & \text{for } i \neq j \\ x_k \longmapsto x_k & \text{for } k \neq i. \end{cases}$$

The group

$$E\Pi A_n = \langle \mu_{ij} \mid 1 \leq i \neq j \leq n \rangle$$

is called the elementary palindromic automorphism group of F_n, and the group

$$ES_n = \langle t_i, \alpha_{j,j+1} \mid 1 \leq i \leq n,\ 1 \leq j \leq n-1 \rangle$$

is called the extended symmetric group. In [1], Collins showed that

$$\Pi A_n \cong E\Pi A_n \rtimes ES_n$$

for $n \geq 2$. Here, ES_n acts on $E\Pi A_n$ by conjugation given by the following rules:

$$t_i \mu_{ij} t_i = \mu_{ij}^{-1},\ t_j \mu_{ij} t_j = \mu_{ij}^{-1},$$
$$t_k \mu_{ij} t_k = \mu_{ij} \text{ for } k \neq i, j,$$
$$\alpha \mu_{ij} \alpha = \mu_{\alpha(i)\alpha(j)} \text{ for } \alpha \in \{\alpha_{1,2}, \alpha_{2,3}, \ldots, \alpha_{n-1,n}\}.$$

In [1], Collins also showed that a set of defining relations for $E\Pi A_n$ is

$$\mu_{ij}\mu_{kl} = \mu_{kl}\mu_{ij},$$
$$\mu_{ik}\mu_{jk} = \mu_{jk}\mu_{ik},$$
$$\mu_{ik}\mu_{jk}\mu_{ij} = \mu_{ij}\mu_{jk}\mu_{ik}^{-1}.$$

In the same paper, Collins conjectured that $E\Pi A_n$ is torsion free for each $n \geq 2$. Using geometric techniques, Glover and Jensen [4] proved this conjecture and also calculated the