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Partial compact quantum groups

Kenny De Commer [a,*], Thomas Timmermann [b]

[a] *Department of Mathematics, Vrije Universiteit Brussel, VUB, B-1050 Brussels, Belgium*
[b] *University of Münster, Einsteinstrasse 62, 48149 Münster, Germany*

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ABSTRACT

Compact quantum groups of face type, as introduced by Hayashi, form a class of quantum groupoids with a classical, finite set of objects. Using the notions of weak multiplier bialgebras and weak multiplier Hopf algebras (resp. due to Böhm–Gómez-Torrecillas–López-Centella and Van Daele–Wang), we generalize Hayashi's definition to allow for an infinite set of objects, and call the resulting objects partial compact quantum groups. We prove a Tannaka–Kreĭn–Woronowicz reconstruction result for such partial compact quantum groups using the notion of partial fusion C*-categories. As examples, we consider the dynamical quantum $SU(2)$-groups from the point of view of partial compact quantum groups.

Introduction

The concept of *face algebra* was introduced by T. Hayashi in [13], motivated by the theory of solvable lattice models in statistical mechanics. It was further studied in [14–20], where for example associated *-structures and a canonical Tannaka duality were developed. This Tannaka duality allows one to construct a canonical face algebra from

* Corresponding author.
E-mail addresses: kenny.de.commer@vub.ac.be (K. De Commer), timmermt@uni-muenster.de (T. Timmermann).

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any (finite) fusion category. For example, a face algebra can be associated to the fusion category of a quantum group at root unity, for which no genuine quantum group implementation can be found.

In [32,36,37], it was shown that face algebras are particular kinds of $\times_R$-algebras [40] and of weak bialgebras [5,3,31]. More intuitively, they can be considered as quantum groupoids with classical, finite object set. In this article, we want to extend Hayashi's theory by allowing an *infinite* (but still discrete) object set. This requires passing from weak bialgebras to weak *multiplier* bialgebras [4]. At the same time, our structures admit a piecewise description by what we call a *partial bialgebra*, which is more in the spirit of Hayashi's original definition. In the presence of an antipode, an invariant integral and a compatible *-structure, we call our structures *partial compact quantum groups*.

The passage to the infinite object case requires extra arguments at certain points, as one has to impose the proper finiteness conditions on associated structures. However, once all conditions are in place, many of the proofs are similar in spirit to the finite object case.

Our main result is a Tannaka–Kreĭn–Woronowicz duality result which states that partial compact quantum groups (with finite hyperobject set) are, up to the appropriate notion of equivalence, in one-to-one correspondence with *concrete semisimple rigid tensor* C*-*categories*. Here we do not assume the unit of the tensor C*-category to be irreducible – this situation can also be dealt with using the notion of C*-bicategory introduced in [21]. By a *concrete* tensor C*-category we mean a tensor C*-category realized inside a category of (locally finite-dimensional) bigraded Hilbert spaces. Of course, Tannaka reconstruction is by now a standard procedure. For closely related results most relevant to our work, we mention [48,35,16,33,12,39,34,9,29] as well as the surveys [22] and [30, Section 2.3].

As an application, we generalize Hayashi's Tannaka duality [16] (see also [33]) by showing that any module C*-category over a semisimple rigid tensor C*-category has an associated canonical partial compact quantum group. By the results of [9], such data can be produced from ergodic actions of compact quantum groups. In particular, we consider the case of ergodic actions of $SU_q(2)$ for q a non-zero real. This will allow us to show that the construction of [14] generalizes to produce partial compact quantum group versions of the dynamical quantum $SU(2)$-group [11,24], see also [38] and the references therein. This construction will provide the right setting for the operator algebraic versions of these dynamical quantum $SU(2)$-groups, which was the main motivation for writing this paper. These operator algebraic details will be studied elsewhere [7].

The precise layout of the paper is as follows.

The *first section* introduces the basic theory of the structures which we will be concerned with. We introduce the notions of a *partial bialgebra*, *partial Hopf algebra* and *partial compact quantum group*, and show how they are related to the notion of a weak multiplier bialgebra [4], weak multiplier Hopf algebra [46,45] and compact quantum group of face type [14]. We also briefly recall the notions of *tensor category* and *tensor* C*-*category*.