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Character correspondences in blocks with normal defect groups ☆

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ABSTRACT

In this paper we give an extension of the Glauberman correspondence to certain characters of blocks with normal defect groups.
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1. Introduction

Let p be a fixed prime number. The Glauberman correspondence appears in many key places in the character theory of finite groups, specially in those connecting global and local representations. In what perhaps constitutes the most relevant case, the Glauberman correspondence asserts that if a finite p-group P acts as automorphisms on a finite group K of order not divisible by p, then there is a natural bijection between $\mathrm{Irr}_P(K)$, the P-invariant irreducible characters of K, and $\mathrm{Irr}(C)$, the irreducible characters of the fixed point subgroup $C = \mathbf{C}_K(P)$. (In fact, the Glauberman correspondence

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is defined whenever P is a solvable group acting coprimely on a group G, see Chapter 13 of [Is]. Later on, M. Isaacs removed the hypothesis of P being solvable in [Is2].) The Glauberman correspondence (in the case where a p-group is acting) was noticed to be a consequence of the Brauer correspondence between blocks by J. Alperin in [A].

In 1980, E.C. Dade (using the Brauer correspondence) extended the Glauberman correspondence to a correspondence between certain defect zero characters of $\mathrm{Irr}_P(K)$ and $\mathrm{Irr}(C)$, whenever a group K is acted by a p-group P [D]. This was also discovered independently by H. Nagao (see Theorem (5.12.1) in [NT]), and we have called this the Dade–Glauberman–Nagao correspondence (DGN) in [NaTi].

While the Glauberman correspondence is a key ingredient in the reduction of the McKay conjecture to simple groups, and the Dade–Glauberman–Nagao correspondence plays an important role in a reduction of the Alperin Weight Conjecture [NaTi], it is somewhat remarkable that we need now an extension of the Dade–Glauberman–Nagao correspondence in order to carry out a reduction to simple groups of the unproven half of Brauer's Height Zero conjecture [NS].

To state our Theorem A below, we remind the reader that if G is a finite group, $N \lhd G$, then an irreducible character $\chi \in \mathrm{Irr}(G)$ has **relative p-defect zero** with respect to N (or that χ has **N-relative p-defect zero**) if

$$\big(\chi(1)/\theta(1)\big)_p = |G/N|_p,$$

where $\theta \in \mathrm{Irr}(N)$ is any irreducible constituent of the restriction χ_N. (Let us briefly mention here that the significance of relative p-defect zero characters, or, in their terminology, of *N-relatively projective characters*, was already pointed out by B. Külshammer and G.R. Robinson in the remarkable paper [KR], and that we shall be using here some techniques introduced by them.)

We need some new notation in order to state our main result. Let G be a finite group. If a p-subgroup P of G normalizes some subgroup $K \leqslant G$, then we denote by $\mathrm{Bl}(K|P)$ the set of p-blocks b of K such that the unique block of KP covering b has defect group P. (If P is contained in K, then $\mathrm{Bl}(K|P)$ becomes the set of blocks of K with defect group P.) If $\tau \in \mathrm{Irr}(G)$, then $\mathrm{bl}(\tau)$ is the p-block of G that contains τ. If $D \lhd G$ and $\mu \in \mathrm{Irr}(D)$, then G_μ is the stabilizer of μ in G, and if $\chi \in \mathrm{Irr}(G)$ lies over μ, then $\chi_\mu \in \mathrm{Irr}(G_\mu)$ is the Clifford correspondent of χ over μ.

Theorem A. *Let G be a finite group, and let p be a prime. Suppose that $K \lhd G$, P is a p-subgroup of $G = KP$ and $P \cap K = D \lhd G$. Let $C = \mathbf{N}_K(P)$. Then:*

(a) *There is a natural bijection $'$: $\mathrm{Bl}(K|P) \to \mathrm{Bl}(C|P)$.*
(b) *If $b \in \mathrm{Bl}(K|P)$, then there is a natural bijection*

$$' : \mathrm{Irr}_P(b) \to \mathrm{Irr}_P\big(b'\big),$$

where $\mathrm{Irr}_P(b)$ are the P-invariant irreducible characters of b.
(c) *If $\eta \in \mathrm{Irr}_P(b)$, then there is a unique C-conjugacy class of P-invariant irreducible constituents $\mu \in \mathrm{Irr}(D)$ of the restriction η_D such that $\mathrm{bl}(\eta_\mu) \in \mathrm{Bl}(K_\mu|P)$. Furthermore, η' is the unique irreducible D-relative p-defect zero constituent of η_C with multiplicity not divisible by p lying over μ.*
(d) *If $\eta \in \mathrm{Irr}_P(b)$, then*

$$\big[\eta_C, \eta'\big] \equiv \pm 1 \mod p.$$

Of course, when K is a p'-group, then the correspondence in Theorem A is the p-group case of the Glauberman correspondence, and when $D = 1$, this is the extension by Dade and Nagao. Also the case where $D \in \mathrm{Syl}_p(K)$ was recently obtained in [IN], in a totally different context.

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