Journal of Functional Analysis 270 (2016) 2249–2267

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Infinity Laplacian equation with strong absorptions

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A R T I C L E   I N F O

Article history:
Received 15 November 2014
Accepted 17 December 2015
Available online 12 January 2016
Communicated by C. De Lellis

MSC:
35J60
35B65

Keywords:
Reaction–diffusion equations
Infinity Laplacian
Regularity

A B S T R A C T

We study regularity properties of solutions to reaction–diffusion equations ruled by the infinity Laplacian operator. We focus our analysis in models presenting plateaus, i.e. regions where a non-negative solution vanishes identically. We obtain sharp geometric regularity estimates for solutions along the boundary of plateaus sets. In particular we show that the $(n-\epsilon)$-Hausdorff measure of the plateaus boundary is finite, for a universal number $\epsilon > 0$.

1. Introduction

The mathematical analysis of problems involving the infinity Laplacian operator,

$$\Delta_\infty u := \sum_{i,j} \partial_i u \partial_{ij} u \partial_j u = (Du)^T D^2 u \, Du, \tag{1.1}$$

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http://dx.doi.org/10.1016/j.jfa.2015.12.012
0022-1236/

constitutes a beautiful chapter of the modern theory of partial differential equations, yet far from its denouement. The systematic study of problems involving the infinity Laplacian operator has been originated by the pioneering works of G. Aronsson [1,2]. The initial purpose of this line of research is to answer the following natural question: given a bounded domain $O \subset \mathbb{R}^n$ and a Lipschitz function $g\colon \partial O \to \mathbb{R}$, find its best Lipschitz extension, f, in the sense that it agrees with g on the boundary and for any $O' \Subset O$, if $f = h$ on $\partial O'$, then $\|f\|_{\mathrm{Lip}(O')} \le \|h\|_{\mathrm{Lip}(O')}$. Such a function f is said to be an absolutely minimizing Lipschitz extension of g in O. Jensen in [10] has proven that a function in an absolutely minimizing Lipschitz extension if, and only if, it is a viscosity solution to the homogeneous equation $\Delta_\infty u = 0$. That is, the infinity Laplacian rules the Euler–Lagrange equation associated to this L^∞ minimization problem.

Through the years, several different applications of the infinity Laplacian theory emerged in the literature [5,14,4], just to cite few. We refer to [3] for an elegant discussion on the theory of absolutely minimizing Lipschitz extensions.

While, existence and uniqueness of viscosity solution to the homogeneous Dirichlet problem $\Delta_\infty h = 0$, in O, $u = g$, on ∂O is nowadays fairly well established, obtaining improved regularity estimates for infinity harmonic functions remains a major open issue in the theory of nonlinear partial differential equations. The example of Aronsson

$$h(x, y) = x^{4/3} - y^{4/3}$$

hints out to one of the most famous conjectures in this field: the first derivatives of infinity harmonic functions should be Hölder continuous with optimal exponent $\frac{1}{3}$. The best results known up to date are due to Evans and Savin [8], who proved that infinity harmonic functions in the plane are of class $C^{1,\alpha}$, for some $0 < \alpha \ll 1$, see also [17], and to Evans and Smart [9], who obtained everywhere differentiability for infinity harmonic functions in any dimension.

The theory of inhomogeneous infinity Laplacian equations $\Delta_\infty u = f(X)$ is more recent and subtle. Lu and Wang in [13] has proven existence and uniqueness of continuous viscosity solutions to the Dirichlet problem

$$\begin{cases} \Delta_\infty u = f(X) & \text{in } O \\ \quad\;\; u = g & \text{on } \partial O, \end{cases} \tag{1.2}$$

provided the source function f does not change sign, i.e. either $\inf f > 0$ or else $\sup f < 0$. Uniqueness may fail if such a condition is violated [13, Appendix A]. While Lipschitz estimates and everywhere differentiability also hold for a function whose infinity Laplacian is bounded in the viscosity sense, see [12], no further regularity is so far known for inhomogeneous equations.

This current work is devoted to the study of reaction–diffusion models ruled by the infinity Laplacian operator. Namely, for $\lambda > 0$ and $0 \le \gamma < 3$, let

$$\mathscr{L}_\infty^\gamma v := \Delta_\infty v - \lambda (v^+)^\gamma \tag{1.3}$$