



Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa



Expected volume and Euler characteristic of random submanifolds



Thomas Letendre

École Normale Supérieure de Lyon, Unité de Mathématiques Pures et Appliquées, UMR CNRS 5669, 46 allée d'Italie, 69634 Lyon Cedex 07, France

ARTICLE INFO

Article history:

Received 11 March 2015
Accepted 11 January 2016
Available online 26 February 2016
Communicated by B. Chow

MSC:

14P25
32A25
34L20
60D05
60G60

Keywords:

Euler characteristic
Riemannian random wave
Random polynomial
Real projective manifold

ABSTRACT

In a closed manifold of positive dimension n , we estimate the expected volume and Euler characteristic for random submanifolds of codimension $r \in \{1, \dots, n\}$ in two different settings. On one hand, we consider a closed Riemannian manifold and some positive λ . Then we take r independent random functions in the direct sum of the eigenspaces of the Laplace–Beltrami operator associated to eigenvalues less than λ and consider the random submanifold defined as the common zero set of these r functions. We compute asymptotics for the mean volume and Euler characteristic of this random submanifold as λ goes to infinity. On the other hand, we consider a complex projective manifold defined over the reals, equipped with an ample line bundle \mathcal{L} and a rank r holomorphic vector bundle \mathcal{E} that are also defined over the reals. Then we get asymptotics for the expected volume and Euler characteristic of the real vanishing locus of a random real holomorphic section of $\mathcal{E} \otimes \mathcal{L}^d$ as d goes to infinity. The same techniques apply to both settings.

© 2016 Elsevier Inc. All rights reserved.

Contents

1. Introduction 3048

E-mail address: thomas.letendre@ens-lyon.fr.

2.	Random submanifolds	3053
2.1.	General setting	3053
2.2.	The incidence manifold	3054
2.3.	The covariance kernel	3055
2.4.	Random jets	3056
2.5.	Riemannian random waves	3058
2.6.	The real algebraic setting	3059
3.	Estimates for the covariance kernels	3062
3.1.	The spectral function of the Laplacian	3063
3.2.	Hörmander–Tian peak sections	3064
3.3.	The Bergman kernel	3066
4.	An integral formula for the Euler characteristic of a submanifold	3069
4.1.	The algebra of double forms	3069
4.2.	The Chern–Gauss–Bonnet theorem	3071
4.3.	The Gauss equation	3072
4.4.	An expression for the second fundamental form	3073
5.	Proofs of the main theorems	3075
5.1.	The Kac–Rice formula	3075
5.2.	Proof of Theorem 1.1	3076
5.3.	Proof of Theorem 1.3	3079
5.4.	Proof of Theorem 1.2	3081
5.5.	Proof of Theorem 1.4	3089
6.	Two special cases	3091
6.1.	The flat torus	3091
6.2.	The projective space	3093
	Acknowledgments	3095
	Appendix A. Concerning Gaussian vectors	3095
	A.1. Variance and covariance as tensors	3095
	A.2. Gaussian vectors	3098
	Appendix B. Proof of Proposition 5.8	3100
	Appendix C. Proof of the Kac–Rice formula	3105
	C.1. The coarea formula	3105
	C.2. The double-fibration trick	3105
	C.3. Proof of Theorem 5.3	3108
	References	3108

1. Introduction

Zeros of random polynomials were first studied by Bloch and Pòlya [6] in the early 30s. About ten years later, Kac [22] obtained a sharp asymptotic for the expected number of real zeros of a polynomial of degree d with independent standard Gaussian coefficients, as d goes to infinity. This was later generalized to other distributions by Kostlan in [23]. In particular, he introduced a normal distribution on the space of homogeneous polynomials of degree d — known as the Kostlan distribution — which is more geometric, in the sense that it is invariant under isometries of $\mathbb{C}\mathbb{P}^1$. Bogomolny, Bohigas and Leboeuf [7] showed that this distribution corresponds to the choice of d independent roots, uniformly distributed in the Riemann sphere.

In higher dimension, the question of the number of zeros can be generalized in at least two ways. What is the expected volume of the zero set? And what is its expected Euler characteristic? More generally, one can ask what are the expected volume and

Download English Version:

<https://daneshyari.com/en/article/6415055>

Download Persian Version:

<https://daneshyari.com/article/6415055>

[Daneshyari.com](https://daneshyari.com)