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Expected volume and Euler characteristic of random submanifolds

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In a closed manifold of positive dimension n, we estimate the expected volume and Euler characteristic for random submanifolds of codimension $r \in \{1, \dots, n\}$ in two different settings. On one hand, we consider a closed Riemannian manifold and some positive λ. Then we take r independent random functions in the direct sum of the eigenspaces of the Laplace–Beltrami operator associated to eigenvalues less than λ and consider the random submanifold defined as the common zero set of these r functions. We compute asymptotics for the mean volume and Euler characteristic of this random submanifold as λ goes to infinity. On the other hand, we consider a complex projective manifold defined over the reals, equipped with an ample line bundle $\mathcal{L}$ and a rank r holomorphic vector bundle $\mathcal{E}$ that are also defined over the reals. Then we get asymptotics for the expected volume and Euler characteristic of the real vanishing locus of a random real holomorphic section of $\mathcal{E} \otimes \mathcal{L}^d$ as d goes to infinity. The same techniques apply to both settings.

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1. Introduction

Zeros of random polynomials were first studied by Bloch and Pòlya [6] in the early 30s. About ten years later, Kac [22] obtained a sharp asymptotic for the expected number of real zeros of a polynomial of degree d with independent standard Gaussian coefficients, as d goes to infinity. This was later generalized to other distributions by Kostlan in [23]. In particular, he introduced a normal distribution on the space of homogeneous polynomials of degree d — known as the Kostlan distribution — which is more geometric, in the sense that it is invariant under isometries of $\mathbb{CP}^1$. Bogomolny, Bohigas and Leboeuf [7] showed that this distribution corresponds to the choice of d independent roots, uniformly distributed in the Riemann sphere.

In higher dimension, the question of the number of zeros can be generalized in at least two ways. What is the expected volume of the zero set? And what is its expected Euler characteristic? More generally, one can ask what are the expected volume and