Journal of Functional Analysis 270 (2016) 2383–2414

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Regularity for weak solutions to nondiagonal quasilinear degenerate elliptic systems

Yan Dong, Pengcheng Niu *

Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an, Shaanxi, 710129, China

A R T I C L E I N F O

Article history:
Received 11 January 2014
Accepted 6 February 2016
Available online 19 February 2016
Communicated by M. Hairer

MSC:
35J48
35D30

Keywords:
Nondiagonal quasilinear degenerate elliptic system
Hörmander's vector fields
Regularity

A B S T R A C T

The aim of this paper is to establish regularity for weak solutions to the nondiagonal quasilinear degenerate elliptic systems related to Hörmander's vector fields, where the coefficients are bounded with vanishing mean oscillation. We first prove L^p $(p \geq 2)$ estimates for gradients of weak solutions by using a priori estimates and a known reverse Hölder inequality, and consider regularity to the corresponding nondiagonal homogeneous degenerate elliptic systems. Then we get higher Morrey and Campanato estimates for gradients of weak solutions to original systems and Hölder estimates for weak solutions.

1. Introduction

Regularity for solutions to elliptic systems in Euclidean spaces has been studied by many authors. Campanato in [2] has obtained gradient estimates for weak solutions to

* Corresponding author.
E-mail address: pengchengniu@nwpu.edu.cn (P. Niu).

http://dx.doi.org/10.1016/j.jfa.2016.02.006

linear elliptic system with discontinuous coefficients. For related articles, we quote [1,14] and the references therein.

Morrey estimates for uniformly elliptic systems by applying Campanato's technique were derived by Huang in [18]. Zheng and Feng in [28] established Hölder estimates for weak solutions to quasilinear elliptic systems by the reverse Hölder inequality and Dirichlet growth theorem, where the coefficients belong to $L^\infty \cap VMO$ and low terms satisfy controlled growth conditions. For the following second order quasilinear elliptic systems

$$-D_\alpha a_i^\alpha(x,u,Du) = a_i(x,u,Du),$$

where $a_i^\alpha(x,u,Du) = A_{ij}^{\alpha\beta}(x)D_\beta u^j + g_i^\alpha(x,u,Du)$, $A_{ij}^{\alpha\beta} \in C^{0,\alpha}$, Daněček in [4] proved Morrey and Campanato estimates with $p = 2$ for weak solutions. When the coefficient $A_{ij}^{\alpha\beta}$ belongs to $L^\infty(\Omega)\cap\mathcal{L}_\phi(\Omega)$ (where $\phi = \frac{1}{1+|\ln r|}$), $A_{ij}^{\alpha\beta}$ belongs to $VMO(\Omega)\cap L^\infty(\Omega)$, or $A_{ij}^{\alpha\beta}$ is bounded and belongs to Campanato spaces, Daněček and Viszus in [5–7] achieved similar estimates. The L^p estimate for weak solutions to divergence linear elliptic systems was given in Chiarenza, Franciosi and Frasca [3] by representation formula.

To nondiagonal elliptic systems, Kawohl in [19] investigated Hölder continuity for bounded weak solutions to quasilinear elliptic systems if the Liouville type property is satisfied. Hölder regularity for weak solutions to nondiagonal systems with low terms satisfying natural conditions was gained by Wiegner [25]. For more results also see [10, 15,21,22,24,30] and the references therein.

Regularity of degenerate elliptic systems formed by Hörmander's vector fields [17] has received wide attention in recent years. Di Fazio and Fanciullo in [8] proved Morrey estimates for weak solutions to linear degenerate elliptic systems with $p = 2$. Dong and Niu in [9] showed Morrey estimates for linear degenerate elliptic systems if $p \geq 2$. For nonlinear systems, Xu and Zuily in [26] dealt with the interior regularity of weak solutions to quasilinear degenerate elliptic systems with the low term satisfying the natural condition. Gao, Niu and Wang in [11] settled partial Hölder regularity for weak solutions to degenerate quasilinear elliptic systems with the coefficients belonging to $VMO \cap L^\infty$ and the low term satisfying the natural condition. We mention that those systems in [8,9,11,26] are all diagonal.

Up to now, we do not see any regularity result to nondiagonal degenerate elliptic systems. This leaves several doubts: whether do these systems have regularity? What is the kind of regularity if they have? These are what we will answer in this paper. Concretely, we consider the following nondiagonal quasilinear degenerate elliptic system

$$-X_\alpha^*(a_{ij}^{\alpha\beta}(x,u)X_\beta u^j) = g_i(x,u,Xu) - X_\alpha^* f_i^\alpha(x,u,Xu), \tag{1.1}$$

where $\alpha,\beta = 1,2,\dots,q$ $(q \leq n)$, $i,j = 1,2,\dots,N$, $X_\alpha = \sum\limits_{k=1}^{n} b_{\alpha k}(x)\frac{\partial}{\partial x_k}$ $(b_{\alpha k}(x) \in C^\infty(\Omega))$ are real smooth vector fields in a neighborhood $\tilde{\Omega}$ of some bounded domain