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Global structure of radial positive solutions for a prescribed mean curvature problem in a ball

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ABSTRACT

In this paper, we are concerned with the global structure of radial positive solutions of boundary value problem

$$\operatorname{div}\big(\phi_N(\nabla v)\big) + \lambda f(|x|, v) = 0 \quad \text{in} \quad B(R), \quad v = 0 \quad \text{on} \quad \partial B(R),$$

where $\phi_N(y) = \frac{y}{\sqrt{1-|y|^2}}$, $y \in \mathbb{R}^N$, λ is a positive parameter, $B(R) = \{x \in \mathbb{R}^N : |x| < R\}$, and $|\cdot|$ denotes the Euclidean norm in $\mathbb{R}^N$. All results, depending on the behavior of nonlinear term f near 0, are obtained by using global bifurcation techniques.

1. Introduction

In this paper we are concerned with the global structure of radial positive solutions of Dirichlet problem in a ball, associated to mean curvature operator in flat Minkowski space

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$$\mathbb{L}^{N+1} := \{(x,t) : x \in \mathbb{R}^N, t \in \mathbb{R}\}$$

endowed with the Lorentzian metric

$$\Sigma_{j=1}^N (dx_j)^2 - (dt)^2,$$

where (x,t) are the canonical coordinates in $\mathbb{R}^{N+1}$.

It is known (see e.g. [1,4,11,24,27]) that the study of spacelike submanifolds of codimension one in $\mathbb{L}^{N+1}$ with prescribed mean extrinsic curvature leads to Dirichlet problems of the type

$$\mathcal{M}v = H(x,v) \quad \text{in} \quad \Omega, \qquad v = 0 \quad \text{on } \partial\Omega, \tag{1.1}$$

where

$$\mathcal{M}v = \operatorname{div}\Big(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\Big),$$

Ω is a bounded domain in $\mathbb{R}^N$ and the nonlinearity $H : \Omega \times \mathbb{R} \to \mathbb{R}$ is continuous.

The starting point of this type of problems is the seminal paper [11] which deals with entire solutions of $\mathcal{M}v = 0$. The equation $\mathcal{M}v =$ constant is then analyzed in [27], while $\mathcal{M}v = f(v)$ with a general nonlinearity f is considered in [9]. On the other hand, in [16] the author considered the Neumann problem

$$\mathcal{M}v = \kappa v + \lambda \quad \text{in} \quad B(R), \qquad \partial_\nu v = 0 \quad \text{on } \partial B(R),$$

where $B(R) = \{x \in \mathbb{R}^N : |x| < R\}$, $\lambda \neq 0$, $\kappa > 0$, $\mu \in [0,1)$ and $N = 2$. More general sign changing nonlinearities are studied in [6].

If H is bounded, then it has been shown by Bartnik and Simon [4] that (1.1) has at least one solution $u \in C^1(\Omega) \cap W^{2,2}(\Omega)$. Also, when Ω is a ball or an annulus in $\mathbb{R}^N$ and the nonlinearity H has a radial structure, then it has been proved in [5] that (1.1) has at least one classical radial solution. This can be seen as a *universal* existence result for the above problem in the radial case. On the other hand, in this context the existence of positive solutions has been scarcely explored in the related literature, see [7,8].

Very recently, Bereanu, Jebelean and Torres [7] used Leray–Schauder degree arguments and critical point theory for convex, lower semicontinuous perturbations of C^1-functionals, proved existence of classical positive radial solutions for Dirichlet problems

$$\mathcal{M}v + f(|x|, v) = 0 \quad \text{in} \ B(R), \quad v = 0 \quad \text{on} \ \partial B(R), \tag{1.2}$$

under the condition

(H_1) $f : [0,R] \times [0,\alpha) \to \mathbb{R}$ is a continuous function, with $0 < \alpha \leq \infty$ and such that $f(r,s) > 0$ for all $(r,s) \in (0,R] \times (0,\alpha)$.