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A universal hypercyclic representation [☆]

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ABSTRACT

For any countable group, and also for any locally compact second countable, compactly generated topological group, G, we show the existence of a "universal" hypercyclic (i.e. topologically transitive) representation on a Hilbert space, in the sense that it simultaneously models every possible ergodic probability measure preserving free action of G.

Contents

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1. Introduction

A bounded linear operator S on a Banach space B is said to be *hypercyclic* if there are vectors $v \in B$ such that the sequence $\{S^n v\}_{n\geq 0}$ is dense in B. It is called *frequently hypercyclic* if there are vectors v such that for any non-empty open set $U \subset B$ one has

$$\liminf_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \mathbf{1}_U(S^n v) > 0. \tag{1.1}$$

One way to get such a property is for there to be a globally supported S-invariant probability measure μ on B such that the dynamical system (B, S, μ) is ergodic. In that case (1.1) will hold for μ-a.e. v by Birkhoff's ergodic theorem. We shall call S *universal* if for every ergodic probability measure preserving dynamical system $\mathbf{X} = (X, \mathcal{B}, \mu, T)$, there exists an S-invariant probability measure ν on B which is positive on every nonempty open subset of B and such that the dynamical systems $\mathbf{X}$ and $(B, \mathit{Borel}(B), \nu, S)$ are isomorphic. More generally, we have the following definition.

Definition 1.1. For a topological group G, a linear representation as operators on a Banach space B will be called *universal* if for every ergodic probability measure preserving free G-action $\mathbf{X} = (X, \mathcal{B}, \mu, \{T_g\}_{g\in G})$, there exists an $\{S_g\}_{g\in G}$-invariant probability measure ν on B which is positive on every nonempty open subset of B and such that the G-actions $\mathbf{X}$ and $(B, \mathit{Borel}(B), \nu, \{S_g\}_{g\in G})$ are isomorphic.

In this work we show the existence of a universal representation on Hilbert space for the following classes of groups.

1. All countable discrete groups.
2. All locally compact, second countable, compactly generated groups.
3. Groups G of the form $G = \cup_{n=1}^{\infty} K_n$ where $K_1 < K_2 < \cdots$ is an increasing sequence of compact open subgroups.

The precise statement for a countable infinite group is as follows:

Theorem 1.2. *Let G be a countable infinite discrete group. There exists a faithful representation of G, $g \mapsto S_g$, as a group of bounded linear operators on a separable Hilbert space H which is universal.*