



Spectra of weighted composition operators with automorphic symbols

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Abstract

Let φ be an automorphism of the open unit disc \mathbb{D} . For such φ , we investigate the spectra of invertible weighted composition operators uC_φ acting on a wide class of analytic function spaces; this class contains, for example, Hardy spaces $H^p(\mathbb{D})$, weighted Bergman spaces $A_\alpha^p(\mathbb{D})$, and weighted Banach spaces of H^∞ -type. We present new techniques for deducing the spectrum and for calculating the spectral radius of uC_φ . We also characterize the Fredholmness of weighted composition operators on $H^p(\mathbb{D})$ and $A_\alpha^p(\mathbb{D})$.

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1. Introduction

We denote by $H(\mathbb{D})$ the family of all analytic functions on the open unit disc \mathbb{D} of the complex plane \mathbb{C} . Let $\varphi : \mathbb{D} \rightarrow \mathbb{D}$ be an analytic map and $u \in H(\mathbb{D})$. These maps induce via composition and multiplication a linear *weighted composition operator* uC_φ which is defined on $H(\mathbb{D})$ by

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$(uC_\varphi)f = u(f \circ \varphi)$. There are two particularly interesting special cases of such operators: on one hand, taking $u = 1$ gives the *composition operator* C_φ , and on the other, putting $\varphi = \text{id}$, the identity function of \mathbb{D} , gives the *multiplication operator* M_u .

Weighted composition operators are fundamental objects of study in analysis that arise naturally in many situations. A classical result due to Forelli [9] states that all surjective isometries of the Hardy space $H^p(\mathbb{D})$, $1 < p < \infty$, $p \neq 2$, are weighted composition operators. Kolasinski [20] gave a characterization of all surjective isometries of the standard weighted Bergman space $A_\alpha^p(\mathbb{D})$ similar to that of Forelli's. Moreover, weighted composition operators have arisen in the study of commutants of multiplication operators, and they play a role in the theory of dynamical systems as well, just to mention a few examples. Good general references of composition operators on classical spaces of analytic functions on the unit disc are the books written by Cowen and MacCluer [6], and Shapiro [26].

The problem of relating operator theoretic properties (e.g., compactness, and spectrum) of uC_φ to function theoretic properties of u and φ has been a subject of great interest for quite some time; see, e.g., [1,2,4,7,19]. For an automorphic symbol φ , Kamowitz [18] determined the spectrum of uC_φ on the disc algebra $A(\mathbb{D})$ and, more recently, Gunatillake [14] carried out a similar project for invertible weighted composition operators on the Hardy–Hilbert space $H^2(\mathbb{D})$. For a non-automorphic symbol φ with a fixed point in \mathbb{D} , Aron and Lindström [1] completely described the spectrum of a weighted composition operator uC_φ acting on the weighted Banach spaces of H^∞ -type. In the case of a composition operator C_φ acting on $H^2(\mathbb{D})$, Cowen proved in the remarkable paper [5] (see also [6]) several deep and interesting results concerning the spectrum and the essential spectrum of C_φ .

In this paper, by using ideas of Kamowitz and Gunatillake as a starting point, we compute the spectrum of the invertible weighted composition operator uC_φ for an automorphic symbol φ on a wide class of analytic function spaces; this class contains, for example, Hardy spaces, weighted Bergman spaces, and weighted Banach spaces of H^∞ -type. The analysis of the spectral behavior of uC_φ is typically case based, with the cases depending upon the type of the symbol φ , that is, elliptic, parabolic or hyperbolic automorphism. As we will see, parabolic and hyperbolic cases are the most interesting ones having the Denjoy–Wolff point of the automorphism on the boundary of \mathbb{D} . In fact, for these two cases we present new techniques for determining the spectral radius and the spectrum of uC_φ .

One aim of our paper is to complement Gunatillake's work by generalizing his results, since at the end of his paper [14, p. 860] he indicates that he was not able to determine the spectrum of uC_φ on weighted Bergman spaces if φ is either a parabolic or a hyperbolic automorphism of \mathbb{D} . However, our results are not merely generalizations but also improvements of his work, since we get better results for a hyperbolic φ even on the space $H^2(\mathbb{D})$. Our results related to this will be given in Section 4. In Section 3, we characterize the invertibility and Fredholmness of uC_φ on the most important analytic function spaces for a general analytic selfmap φ of \mathbb{D} and arbitrary $u \in H(\mathbb{D})$. An analogous characterization for composition operators on a variety of analytic function spaces have been carried out in [11,10,15,23]. Very recently, Bourdon [3] has obtained more general results on invertibility of weighted composition operators acting on *sets* of analytic functions without norm or linear structure.

Altogether, we show in this paper that it is possible to design a unified approach to determine the spectra of weighted composition operators on a huge class of analytic function spaces. The main results of this paper are stated in **Theorems 4.3 and 4.9**, which concern parabolic and hyperbolic symbols φ , respectively. A summary of our investigation is given at the end of the paper, see **Corollary 5.1**.

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