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On open scattering channels for manifolds with ends

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A B S T R A C T

In the framework of time-dependent geometric scattering theory, we study the existence and completeness of the wave operators for perturbations of the Riemannian metric for the Laplacian on a complete manifold of dimension n. The smallness condition for the perturbation is expressed (intrinsically and coordinate free) in purely geometric terms using the harmonic radius; therefore, the size of the perturbation can be controlled in terms of local bounds on the injectivity radius and the Ricci-curvature. As an application of these ideas we obtain a stability result for the scattering matrix with respect to perturbations of the Riemannian metric. This stability result implies that a scattering channel which interacts with other channels preserves this property under small perturbations.

Contents

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1. Introduction

The first fundamental problem in multi-channel scattering theory is to establish the existence and the (asymptotic) completeness of the wave operators. These questions are currently quite well understood in various situations including the case of the N-body problem in quantum mechanics, multi-channel scattering in perturbed acoustic and electromagnetic wave guides, and scattering on manifolds with ends; cf., e.g., [51,11,47], and the literature discussed at the end of this introduction. Roughly speaking, asymptotic completeness in multi-channel scattering means the following: as time goes to $\pm\infty$, any scattering state decays into a number of states living in subsystems (channels); these subsystems then evolve according to a simpler reference dynamics, like clusters of particles in the quantum mechanical case, radiation and guided modes for perturbed wave guides, and components that travel into the various ends of a manifold. However, given an initial state belonging to a particular channel (as time goes to $-\infty$), asymptotic completeness does not tell us into which channels our state will decay as time goes to $+\infty$, or, put differently, which subsystems will actually be non-zero. We are therefore led to ask which channels are *open* to an initial state belonging to a particular channel in the past ($t \to -\infty$). Clearly, one would expect that two scattering channels will be open to each other unless a particular obstruction prevents the decay from one into the other; put differently, two channels should be open to each other in some generic sense. It appears, though, that there are no general methods in mathematical multi-channel scattering which would allow to prove such a result.

1.1. Open scattering channels

As a first step in the analysis of this issue, the present paper studies the interaction of the channels in geometric scattering theory where the dynamics is given by the Laplacian on a complete n-dimensional Riemannian manifold with a finite number of ends. Any geometric end gives rise to a scattering channel provided the corresponding decoupled part of the Laplacian has a non-zero absolutely continuous part. One of our main results (cf. Theorem 5.1 and Corollary 5.3) roughly says the following: Suppose that the i-th scattering channel is open to the k-th channel in the sense that the channel scattering operator S_{ik} for these channels satisfies