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Purely infinite partial crossed products

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A B S T R A C T

Let $(\mathcal{A}, G, \alpha)$ be a partial dynamical system. We show that there is a bijective correspondence between G-invariant ideals of $\mathcal{A}$ and ideals in the partial crossed product $\mathcal{A} \rtimes_{\alpha,r} G$ provided the action is exact and residually topologically free. Assuming, in addition, a technical condition—automatic when $\mathcal{A}$ is abelian—we show that $\mathcal{A} \rtimes_{\alpha,r} G$ is purely infinite if and only if the positive nonzero elements in $\mathcal{A}$ are properly infinite in $\mathcal{A} \rtimes_{\alpha,r} G$. As an application we verify pure infiniteness of various partial crossed products, including realisations of the Cuntz algebras $\mathcal{O}_n$, $\mathcal{O}_A$, $\mathcal{O}_{\mathbb{N}}$, and $\mathcal{O}_{\mathbb{Z}}$ as partial crossed products.

1. Introduction

In the theory of operator algebras, the crossed product construction has been one of the most important and fruitful tools both to construct examples and to describe the internal structure of operator algebras (in particular the von Neumann algebras).

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Partial actions of a discrete group on C*-algebras and their associated crossed products were gradually introduced in [13] and [34], and since then developed by many authors. Several important classes of C*-algebras have been realised as crossed products by partial actions, including in particular AF-algebras, Cuntz–Krieger algebras, Bunce–Deddens algebras, among others (see for example [5,17,12,14,15,18,25]). The description of C*-algebras as partial crossed products has also proved useful for the computation of their K-theory.

In this paper we pursue the study of partial C*-dynamical systems and their crossed products associated. We begin by recalling (in Section 2 and Appendix A.1) the construction of the partial crossed product $\mathcal{A}\rtimes_{\alpha,r}G$ associated to a *partial action* α (a compatible collection of isomorphisms $\alpha_t : \mathcal{D}_{t^{-1}} \to \mathcal{D}_t$, $t \in G$ of ideals in $\mathcal{A}$).

In Section 3, we study the ideal structure of partial crossed products, generalising the results on C*-dynamical systems obtained by the second author in [41]. Recall that a partial action α on a C*-algebra $\mathcal{A}$ has the *the intersection property* if every nontrivial ideal in $\mathcal{A}\rtimes_{\alpha,r}G$ intersects $\mathcal{A}$ nontrivially. Then (Definition 3.1) we say that a partial C*-dynamical system (A, G, α) has the *residual intersection property* if for every G-invariant ideal $\mathcal{I}$ in $\mathcal{A}$, the induced partial action of G on $\mathcal{A}/\mathcal{I}$ has the intersection property. We establish in Theorem 3.2 a one-to-one correspondence between ideals in $\mathcal{A}\rtimes_{\alpha,r}G$ and G-invariant ideals of $\mathcal{A}$ provided—in fact if and only if—the partial action α is exact and has the residual intersection property. When the partial action α is *minimal* (no nontrivial G-invariant ideals in $\mathcal{A}$), then the *exactness* (any nontrivial G-invariant ideal $\mathcal{I}$ in $\mathcal{A}$ induces a short exact sequence at the level of reduced crossed products) is automatic.

In [18], having defined a topologically free partial action (by partial homeomorphisms) on a locally compact space X, Exel, Laca, and Quigg proved the simplicity of the partial crossed product $C_0(X)\rtimes_{\alpha,r}G$ under the presence of minimality and topological freeness of α. In [32], Lebedev extended the definition of topological freeness to non-commutative partial actions and showed that a topologically free partial action has always *the intersection property*. With Lebedev's result, we recover in Corollary 3.9 the theorem of Exel, Laca, and Quigg.

Theorem 3.2 allows us also to extend Echterhoff and Laca's work on crossed products: We show that the canonical map $\mathcal{J} \mapsto \mathcal{J}\cap\mathcal{A}$ between ideals in $\mathcal{A}\rtimes_{\alpha,r}G$ and G-invariant ideals of $\mathcal{A}$ restricts to a continuous map from the space of prime ideals of $\mathcal{A}\rtimes_{\alpha,r}G$ to the space of G-prime ideals in $\mathcal{A}$. This restriction is a homeomorphism provided α is exact and residually topologically free, where residual topological freeness is an ideal related version of topological freeness. When $\mathcal{A}$ is separable and abelian we show that the space of prime ideals of $\mathcal{A}\rtimes_{\alpha,r}G$ is homeomorphic to the quasi-orbit space of $\operatorname{Prim}\mathcal{A}$.

In Section 4 we generalise some of the main results in [39], by Rørdam and the second named author, to partial C*-dynamical systems. In particular we give sufficient conditions for a partial crossed product to be purely infinite in the sense of Kirchberg and Rørdam. One of the keys assumptions of Theorem 4.2 goes back to Elliott's notion of *proper outerness* (an automorphism α of $\mathcal{A}$ is properly outer if $\|\alpha|_{\mathcal{I}} - \beta\| = 2$ for every