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The p-adic analytic Dedekind sums

Su Hu [a], Min-Soo Kim [b,*]

[a] *Department of Mathematics, South China University of Technology, Guangzhou, Guangdong 510640, China*
[b] *Center for General Education, Kyungnam University, 7(Woryeong-dong) kyungnamdaehak-ro, Masanhappo-gu, Changwon-si, Gyeongsangnam-do 631-701, Republic of Korea*

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A B S T R A C T

In this paper, using Cohen's and Tangedal and Young's theory on the p-adic Hurwitz zeta functions, we construct the analytic Dedekind sums on the p-adic complex plane $\mathbb{C}_p$. We show that these Dedekind sums interpolate Carlitz's higher order Dedekind sums p-adically. From Apostol's reciprocity law for the generalized Dedekind sums, we also prove a reciprocity relation for the special values of these p-adic Dedekind sums. Finally, the parallel results for the analytic Dedekind sums on the p-adic complex plane associated with Euler polynomials have also been given.

* Corresponding author.
E-mail addresses: mahusu@scut.edu.cn (S. Hu), mskim@kyungnam.ac.kr (M.-S. Kim).

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1. Introduction

Throughout this paper, we shall use the following notations.

$\mathbb{C}$	–	the field of complex numbers.
p	–	a prime number.
$\mathbb{Z}_p$	–	the ring of p-adic integers.
$\mathbb{Q}_p$	–	the field of fractions of $\mathbb{Z}_p$.
$\mathbb{C}_p$	–	the completion of a fixed algebraic closure $\overline{\mathbb{Q}}_p$ of $\mathbb{Q}$.
v_p	–	the p-adic valuation of $\mathbb{C}_p$ normalized so that $\lvert p\rvert_p = p^{-v_p(p)} = p^{-1}$.

For arbitrary real numbers x, $[x]$ denotes the greatest integer not exceeding x and $\{x\}$ denotes the fractional part of real number x, thus

$$\{x\} = x - [x]. \tag{1.1}$$

For positive integer h and integer k, the classical Dedekind sum is defined as

$$s(h,k) = \sum_{a \pmod{k}} \left(\left(\frac{ha}{k}\right)\right)\left(\left(\frac{a}{k}\right)\right), \tag{1.2}$$

where $((x))$ denotes

$$((x)) = \begin{cases} x - [x] - \frac{1}{2} & \text{if } x \notin \mathbb{Z}, \\ 0 & \text{if } x \in \mathbb{Z}. \end{cases}$$

This sum appears in the transformation formula of $\log \eta(\tau)$. Here $\eta(\tau)$ is the well-known modular form of weight $\frac{1}{2}$, defined for $\operatorname{Im}\tau > 0$, by

$$\eta(\tau) = e^{\frac{\pi i \tau}{12}} \prod_{n=1}^{\infty} (1 - e^{2\pi i n \tau}).$$

(See [2, p. 52, Theorem 3.4]).

In 1950, Apostol [1] generalized $s(h,k)$ by defining

$$s_m^{(1)}(h,k) = \sum_{a=0}^{k-1} \overline{B}_m\left(\frac{ha}{k}\right)\overline{B}_1\left(\frac{a}{k}\right), \tag{1.3}$$

where $\overline{B}_m(x)$ is the m-th Bernoulli function defined by

$$\overline{B}_m(x) = B_m(\{x\}) \text{ for } m > 1 \quad \text{and} \quad \overline{B}_1(x) = ((x)). \tag{1.4}$$