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# Congruences for $t$ -core partition functions

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## ABSTRACT

Let  $t \geq 2$  be an integer and  $p \geq 5$  be a prime. We prove a conjecture on congruences for  $2^t$ -core partition functions. We also find many new congruences for  $p$ -core partition functions when  $5 \leq p \leq 47$ .

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## 1. Introduction

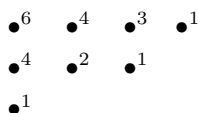
A partition of a positive integer  $n$  is any non-increasing sequence of positive integers whose sum is  $n$ . Given a partition  $[\lambda] = \lambda_1 + \lambda_2 + \cdots + \lambda_k$  of  $n$ , where  $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k$ , the Ferrers–Young diagram of  $[\lambda]$  is an array of nodes with  $\lambda_i$  nodes in the  $i$ th row. The  $(i, j)$  hook is the set of nodes directly below, together with the set of nodes directly to the right of the  $(i, j)$  nodes, as well as the  $(i, j)$  node itself. The hook number, denoted by  $H(i, j)$ , is the total number of nodes on the  $(i, j)$  hook.

For a positive integer  $t$ , a  $t$ -core partition of  $n$  is a partition of  $n$  in which none of the hook numbers are divisible by  $t$ . We denote by  $a_t(n)$  the number of  $t$ -core partitions

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of  $n$ . We illustrate the Ferrers–Young diagram of the partition  $4 + 3 + 1$  of 8 with hook numbers as follows:



It is clear that if  $t \geq 7$ , then the partition  $4 + 3 + 1$  of 8 is a  $t$ -core partition.

$t$ -core partitions are of importance in combinatorial number theory. For example, Garvan, Kim and Stanton [5] used explicit statistics, called cranks, to prove the famous Ramanujan's congruences for the partition function  $p(n)$ : for all nonnegative integers  $n$ ,

$$\begin{aligned} p(5n + 4) &\equiv 0 \pmod{5}, \\ p(7n + 5) &\equiv 0 \pmod{7}, \\ p(11n + 6) &\equiv 0 \pmod{11}. \end{aligned}$$

Moreover, they proved that the generating function for  $a_t(n)$  is

$$\sum_{n=0}^{\infty} a_t(n) q^n = \prod_{n=1}^{\infty} \frac{(1 - q^{tn})^t}{(1 - q^n)}. \quad (1)$$

One can refer to [11] for a different combinatorial proof.  $t$ -core partitions also play important roles in the study of modular representation of symmetric group  $S_n$  [10].

The purpose of this paper is to establish some congruence properties of  $a_t(n)$ . There are numerous beautiful results on congruences for  $a_t(n)$ . For instance, Garvan, Kim and Stanton [5] proved that if  $\alpha$  is a positive integer and  $\ell = 5, 7, 11$ , then

$$a_\ell(\ell^\alpha n - \delta_\ell) \equiv 0 \pmod{\ell^\alpha}$$

for all nonnegative integers  $n$ , where  $\delta_\ell = \frac{\ell^2 - 1}{24}$ . Applying Ramanujan's congruences for the partition function  $p(n)$  modulo powers of 5, 7, 11, Granville and Ono [6] showed that

$$\begin{aligned} a_{5^\alpha}(5^\alpha n - \delta_{5,\alpha}) &\equiv 0 \pmod{5^\alpha}, \\ a_{7^\alpha}(7^\alpha n - \delta_{7,\alpha}) &\equiv 0 \pmod{7^{[\alpha/2]+1}}, \\ a_{11^\alpha}(11^\alpha n - \delta_{11,\alpha}) &\equiv 0 \pmod{11^\alpha}, \end{aligned}$$

where  $\delta_{\ell,\alpha} = \frac{1}{24} \pmod{\ell^\alpha}$ .

When  $t$  is a power of 2, arithmetic properties of  $a_t(n)$  have been studied by the author in [3]. For 4-core, 8-core and 16-core partition functions, Hirschhorn, Kolitsch and

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