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Regular partitions of half-spin geometries

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ABSTRACT

We describe several families of regular partitions of half-spin geometries and determine their associated parameters and eigenvalues. We also give a general method for computing the eigenvalues of regular partitions of half-spin geometries.

1. Introduction

Let q be a prime power and $n \in \mathbb{N} \setminus \{0, 1\}$. Let $Q^+(2n-1, q)$ be a hyperbolic quadric of $\mathrm{PG}(2n-1, q)$ and denote by $\mathcal{M}$ the set of generators of $Q^+(2n-1, q)$, i.e. the set of all subspaces of $Q^+(2n-1, q)$ of maximal (projective) dimension $n-1$. On the set $\mathcal{M}$, an equivalence relation can be defined by calling two generators equivalent whenever they

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intersect in a subspace of even co-dimension. There are two equivalence classes which we will denote by $\mathcal{M}^+$ and $\mathcal{M}^-$.

For every $\epsilon \in \{+, -\}$, the following point-line geometry $HS^\epsilon(2n-1, q)$ can be defined:

- the points of $HS^\epsilon(2n-1, q)$ are the elements of $\mathcal{M}^\epsilon$;
- the lines of $HS^\epsilon(2n-1, q)$ are the subspaces of dimension $n-3$ of $Q^+(2n-1, q)$;
- incidence is reverse containment.

The isomorphic geometries $HS^+(2n-1, q)$ and $HS^-(2n-1, q)$ are called the *half-spin geometries* of $Q^+(2n-1, q)$. We denote any of these geometries by $HS(2n-1, q)$.

The half-spin geometry $HS(3, q)$ is a line containing $q+1$ points, $HS(5, q)$ is isomorphic to $\mathrm{PG}(3, q)$ (regarded as a linear space) and $HS(7, q)$ is isomorphic to the geometry of the points and lines of $Q^+(7, q)$.

In this paper, we study *regular partitions* of half-spin geometries. These are partitions $\mathcal{P} = \{X_1, X_2, \dots, X_{|\mathcal{P}|}\}$ of the point sets for which there are constants a_{ij}, $i, j \in \{1, 2, \dots, |\mathcal{P}|\}$, such that every point $x \in X_i$ is collinear with precisely a_{ij} points of $X_j \setminus \{x\}$. The eigenvalues of the matrix $A_\mathcal{P} = (a_{ij})$ are called the *eigenvalues of the regular partition*, and each of these eigenvalues is also an eigenvalue of the collinearity graph of the geometry.

In this paper, we describe several families of regular partitions of half-spin geometries and determine their parameters and eigenvalues. Regular partitions have already been studied for other families of point-line geometries, like generalized polygons [8] and dual polar spaces [3]. Many regular partitions are associated with nice substructures, and the eigenvalues of these regular partitions might yield information about the structures from which they are derived. Indeed, Proposition 2.1 below shows that these eigenvalues are a helpful tool for determining intersections sizes of combinatorial structures.

This paper is organized as follows. Section 3 contains the main results of this paper. In this section, we describe several families of regular partitions of half-spin geometries, many of which are related to nice geometrical substructures, and mention their parameters and eigenvalues. In Section 4, we describe a general method for computing eigenvalues of regular partitions of half-spin geometries, and in Section 5 we apply this method to compute the eigenvalues of all families of regular partitions described in Section 3. Before we do that, we recall some basic facts about regular partitions, half-spin geometries and hyperbolic dual polar spaces in the next section.

2. Preliminaries

2.1. Regular partitions of point-line geometries

Let X be a nonempty finite set and $R \subseteq X \times X$ a symmetric relation on X.

A partition $\mathcal{P} = \{X_1, X_2, \dots, X_k\}$ of size k of X is called *R-regular* if there exist constants a_{ij}, $i, j \in \{1, 2, \dots, k\}$, such that for every $x \in X_i$, there are precisely a_{ij}