

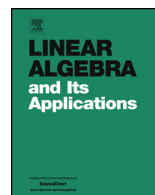


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Some more interplay of the three Kirchhoffian indices



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ABSTRACT

For any simple connected undirected graph, and the random walk on it, we obtain a formula for the sum of all expected hitting times – normalized by the stationary distribution – expressed in terms of the eigenvalues of a certain *modified* Laplacian matrix. This allows us to find lower bounds for these sums of hitting times, as well as new lower bounds for the additive degree-Kirchhoff index, in terms of the multiplicative degree-Kirchhoff index and the Kirchhoff index, that improve other bounds found in the literature.

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1. Introduction

Let $G = (V, E)$ be a finite simple connected graph with vertex set $V = \{1, 2, \dots, n\}$ and degrees d_i for $1 \leq i \leq n$, with δ and Δ the smallest and largest such degrees, respectively, and $d_G = \frac{2|E|}{n}$ the *average degree*. There is a family of resistive – or Kirchhoffian – descriptors $R^f(G)$, studied in Mathematical Chemistry, with the general formula

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$$R^f(G) = \sum_{i < j} f(i, j)R_{i,j}, \tag{1}$$

where R_{ij} is the effective resistance between vertices i and j and $f(i, j)$ is some real function of the vertices. Among these descriptors, the ones that have undergone a more intense scrutiny are the Kirchhoff index $R(G)$, the multiplicative degree-Kirchhoff index $R^*(G)$ and the additive degree-Kirchhoff index $R^+(G)$, defined by (1) when taking $f(i, j) = 1$, $f(i, j) = d_i d_j$ and $f(i, j) = d_i + d_j$, respectively, and introduced in [11,3,6] respectively.

A first observation concerning these indices is that their relationship is very clear when the graphs under consideration are trees. Indeed for any tree T , in the absence of cycles, the effective resistance between any two vertices coincides with the distance between those vertices, and the equalities found in [10] and [5], referred to the so-called Schultz and modified Schultz indices, become the following equalities for the Kirchhoffian indices:

$$R^+(T) = 4R(T) - n(n - 1), \tag{2}$$

and

$$R^*(T) = 4R(T) - (2n - 1)(n - 1), \tag{3}$$

once the distances are replaced with the effective resistances in the original equations. Equalities such as (2) and (3) remain elusive for general graphs and instead a number of inequalities have been found for these resistive descriptors.

Another observation on the Kirchhoffian indices is that two of them have a simple expression in terms of certain eigenvalues, namely (see [7,22], and also [8] and [16] for alternative proofs)

$$R(G) = n \sum_{i=1}^{n-1} \frac{1}{\lambda_i}, \tag{4}$$

for $\lambda_1 \geq \dots \geq \lambda_n = 0$ the eigenvalues of the Laplacian matrix $\mathbf{L} = \mathbf{D} - \mathbf{A}$, where \mathbf{D} is the diagonal matrix with the degrees of the vertices in the diagonal, and \mathbf{A} is the adjacency matrix of G . Likewise (see [3,15])

$$R^*(G) = 2|E| \sum_{i=2}^n \frac{1}{1 - \alpha_i} = 2|E| \sum_{i=1}^{n-1} \frac{1}{\beta_i}, \tag{5}$$

for $1 = \alpha_1 > \alpha_2 \geq \dots \geq \alpha_n \geq -1$ the eigenvalues of the transition probability matrix $\mathbf{P} = \mathbf{D}^{-1}\mathbf{A}$ of the random walk on G , and for $2 \geq \beta_1 \geq \dots \geq \beta_n = 0$ the eigenvalues of the normalized Laplacian matrix $\mathcal{L} = \mathbf{D}^{-1/2}\mathbf{L}\mathbf{D}^{-1/2}$.

These expressions for $R(G)$ and $R^*(G)$ have led to an abundant bibliography of bounds for these indices found by optimizing the summations in (4) and (5) in some way. See

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