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# On the principal eigenvectors of uniform hypergraphs ☆

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A B S T R A C T

Let $\mathcal{A}(H)$ be the adjacency tensor of $r$-uniform hypergraph $H$. If $H$ is connected, the unique positive eigenvector $x = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$ with $||x||_r = 1$ corresponding to spectral radius $\rho(H)$ is called the principal eigenvector of $H$. The maximum and minimum entries of $x$ are denoted by $x_{\max}$ and $x_{\min}$, respectively. In this paper, we investigate the bounds of $x_{\max}$ and $x_{\min}$ in the principal eigenvector of $H$. Meanwhile, we also obtain some bounds of the ratio $x_i/x_j$ for $i$, $j \in [n]$ as well as the principal ratio $\gamma(H) = x_{\max}/x_{\min}$ of $H$. As an application of these results we finally give an estimate of the gap of spectral radii between $H$ and its proper sub-hypergraph $H'$.

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## 1. Introduction

Let $G$ be a simple connected graph, and $A(G)$ be the adjacency matrix of $G$. Perron–Frobenius theorem implies that $A(G)$ has a unique unit positive eigenvector corresponding to spectral radius $\rho(G)$, which is usually called the principal eigenvector of $G$. The

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principal eigenvector plays an important role in spectral graph theory, and there exist some literatures concerning that. In 2000, Papendieck and Recht [16] posed an upper bound for the maximum entry of the principal eigenvector of a graph. Later, Zhao and Hong [23] further investigated bounds for maximum entry of the principal eigenvector of a symmetric nonnegative irreducible matrix with zero trace. In 2007, Cioabă and Gregory [4] improved the bound of Papendieck and Recht [16] in terms of the vertex degree. Das [6,7] obtained bounds for the maximum entry of the principal eigenvector of the signless Laplacian matrix and distance matrix in 2009 and 2011, respectively. Recently, Das et al. [8] determine upper and lower bounds of the maximum entry of the principal eigenvector of the distance signless Laplacian matrix.

Hypergraph is a natural generalization of ordinary graph (see [1]). A *hypergraph* $H = (V, E)$ on $n$ vertices is a set of vertices, say $V = \{1, 2, \cdots, n\}$ and a set of edges, say $E = \{e_1, e_2, \cdots, e_m\}$, where $e_i = \{i_1, i_2, \cdots, i_\ell\}$, $i_j \in [n] := \{1, 2, \cdots, n\}$, $j \in [\ell]$. A hypergraph is called *r-uniform* if every edge contains precisely $r$ vertices. Let $H = (V, E)$ be an $r$-uniform hypergraph on $n$ vertices. The adjacency tensor (see [5]) of $H$ is defined as the order $r$ dimension $n$ tensor $\mathcal{A}(H)$ whose $(i_1 i_2 \cdots i_r)$-entry is

$$(\mathcal{A}(H))_{i_1 i_2 \cdots i_r} = \begin{cases} \dfrac{1}{(r-1)!} & \text{if } \{i_1, i_2, \cdots, i_r\} \in E(H), \\ 0 & \text{otherwise.} \end{cases}$$

Qi [17] and Lim [11] independently introduced the concept of eigenvalues of tensors, from which one can get the definition of the eigenvalues of the adjacency tensor of an $r$-uniform hypergraph (see more in Section 2). Obviously, adjacency tensor is a symmetric nonnegative tensor. The Perron–Frobenius theorem for nonnegative tensors has been established (see [3,21] and the references in them). Based on the Perron–Frobenius theorem there exists a unique positive eigenvector $x$ with $||x||_r = 1$ for $\mathcal{A}(H)$ of a connected $r$-uniform hypergraph $H$. This vector will be called the *principal eigenvector* of $H$. Denote by $x_{\max}$ the maximum entry of $x$ and by $x_{\min}$ the minimum entry of $x$. The *principal ratio*, $\gamma(H)$, of $H$ is defined as $x_{\max}/x_{\min}$.

Recently, Nikiforov [15] presented some bounds on the entry of the principal eigenvector of an $r$-uniform hypergraph (see Section 7 of [15]). Li et al. [9] posed some lower bounds for principal eigenvector of connected uniform hypergraphs in terms of vertex degrees and the number of vertices. In this paper, we generalize some classical bounds on the principal eigenvector to an $r$-uniform hypergraph. In Section 2, we introduce some notations and necessary lemmas. In Section 3, we present some upper bounds on the maximum and minimum entries in the principal eigenvector $x = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$ of a connected $r$-uniform hypergraph $H$. Meanwhile, we also investigate bounds of the ratio $x_i/x_j$ for $i, j \in [n]$ as well as the principal ratio $\gamma(H) = x_{\max}/x_{\min}$ of $H$. Based on these results, in Section 4 we finally give an estimate of the gap of spectral radii between $H$ and its proper sub-hypergraph $H'$.

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