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On eigenspaces of some compound signed graphs

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Dedicated to the memory of Lucia Gionfriddo (1973–2008)

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A B S T R A C T

In the theory of (simple) graphs the concepts of the line and subdivision graph (as compound graphs) are well-known. It is possible to consider them also in the context of (edge) signed graphs. Some relations between the Laplacian spectrum of signed graphs and adjacency spectra of their associated compound (signed) graphs have been recently established in the literature. In this paper, we study the relations between the corresponding eigenspaces.

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1. Introduction

Let $G = (V(G), E(G))$ be a graph (simple, unless otherwise stated) of order $n = |V(G)|$ and size $m = |E(G)|$, and let $\sigma : E(G) \to \{+, -\}$ be a mapping defined on the edge set of G. Then $\Gamma = (G, \sigma)$ is a *signed graph* (or *sigraph*) and G is its *underlying graph*, while σ its *sign function* (or *signature*). Furthermore, it is common to interpret the signs as the integers ± 1. Hence, sometimes signed graphs are treated as weighted graphs, whose (edge) weights belong to $\{+1, -1\}$. An edge e is *positive* (*negative*) if $\sigma(e) = +$ (resp. $\sigma(e) = -$). If all edges in Γ are positive (negative), then Γ is denoted by $(G, +)$ (resp. $(G, -)$).

Most of the concepts defined for graphs are directly extended to signed graphs. For example, the degree of a vertex v in G, denoted by $deg(v)$, is also its degree in Γ. Furthermore, if some subgraph of the underlying graph is observed, then the sign function for the subgraph is the restriction of the original one. Thus, if $v \in V(G)$, then $\Gamma - v$ is the signed subgraph having $G - v$ as the underlying graph, while its signature is the restriction from $E(G)$ to $E(G-v)$ (note, all edges incident to v are deleted). If $U \subset V(G)$ then $\Gamma[U]$ (with underlying graph $G[U]$) denotes the (signed) induced subgraph arising from U, while $\Gamma - U = \Gamma[V(G) \setminus U]$. We also write $\Gamma - \Gamma[U]$ instead of $\Gamma - U$.

A cycle of Γ is said to be *balanced* (or *positive*) if it contains an even number of negative edges. A signed graph is said to be *balanced* if all its cycles are balanced; otherwise, it is *unbalanced*. For $\Gamma = (G, \sigma)$ and $U \subset V(G)$, let Γ^U be the signed graph obtained from Γ by reversing the signature of the edges in the cut $[U, V(G) \setminus U]$, namely, $\sigma_{\Gamma^U}(e) = -\sigma_\Gamma(e)$ for any edge e between U and $V(G) \setminus U$, and $\sigma_{\Gamma^U}(e) = \sigma_\Gamma(e)$ otherwise. The signed graph Γ^U is said to be (signature) *switching equivalent* to Γ, and the corresponding relation is an equivalence relation. So switching equivalent signed graphs can be considered as (switching) isomorphic graphs and their signatures are said to be equivalent. Observe also that switching equivalent graphs have the same set of positive cycles. For a (possibly) complete bibliography on signed graphs, we refer the reader to [19]. For other notation or definitions not given here the reader is referred to [4,5,7] for (unsigned) graph spectra and to [20] for signed graphs.

Signed graphs, as the unsigned ones, can be studied by using matrix theory. If M $(= M(\Gamma))$ is a real and symmetric matrix associated with Γ, then $\det(xI - M)$ is the characteristic polynomial (or M-polynomial) of Γ with respect to M; it is denoted by $\phi_M(x; \Gamma)$. The eigenvalues of M, or equivalently the roots of $\phi_M(x; \Gamma)$, are also called the *eigenvalues of* Γ with respect to M. They are real, since M is real and symmetric. Together with their multiplicities, they comprise the spectrum of Γ (with respect to M) which is denoted by $\hat{\sigma}_M(\Gamma)$. Note that algebraic and geometric multiplicities of any eigenvalue of Γ are the same (since M is real and symmetric). The multiplicity of the eigenvalue μ is denoted by $\mathrm{mult}(\mu; \Gamma)$. A non-zero vector $\mathbf{x}$ satisfying the equation $M\mathbf{x} = \mu\mathbf{x}$, i.e. an *eigenvalue equation*, is the eigenvector (or μ-eigenvector) of M, and also of Γ if it is considered as a vertex labelled signed graph. The eigenspace of M for $\mu \in \hat{\sigma}_M(\Gamma)$ is the set $\mathcal{E}_M(\mu; \Gamma) = \{\mathbf{x} : M\mathbf{x} = \mu\mathbf{x}\}$; it is also an eigenspace of Γ for μ with respect