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Projection on the intersection of convex sets

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In this paper, we give a solution of the problem of projecting a point onto the intersection of several closed convex sets, when a projection on each individual convex set is known. The existing solution methods for this problem are sequential in nature. Here, we propose a highly parallelizable method. The key idea in our approach is the reformulation of the original problem as a system of semi-smooth equations. The benefits of the proposed reformulation are twofold: (a) a fast semi-smooth Newton iterative technique based on Clarke's generalized gradients becomes applicable and (b) the mechanics of the iterative technique is such that an almost decentralized solution method emerges. We proved that the corresponding semi-smooth Newton algorithm converges near the optimal point (quadratically). These features make the overall method attractive for distributed computing platforms, e.g. sensor networks.

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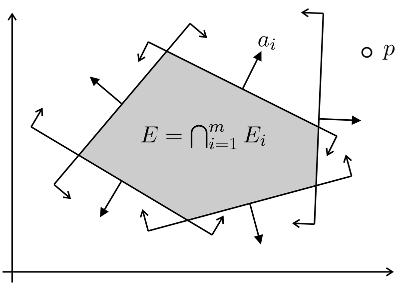
1. Introduction

Let E_i, $i = 1, \ldots, n$, be convex, closed subsets of $\mathbb{R}^m$. Suppose that $E = \bigcap_{i=1}^{n} E_i$ is non-empty. Let $p \in \mathbb{R}^m$ be a point.

By $P_{E_i}(p)$ we denote a projection of a point p to a set E_i. It is well-known that the projection of p on the set E exists and is unique. Thus, we have the following problem:

Problem 1. Find a projection of the point p on the set E.

We assume that the projections of a point p, $P_{E_i}(p)$ on each individual set E_i, $i = 1, \ldots, n$, are well known and easy to handle. However, the projection of a point p on the intersection E $(P_E(p))$ is very hard to compute:



Problem 1 can be alternatively written as the following optimization problem:

$$\underset{\text{subject to } x \in E}{\text{minimize}} \; ||x - p||^2 \tag{1}$$

Problem 1 finds many applications in practice, e.g. in medical imaging, computerized tomography, stat fusion architecture, solving convex problems with strong duality, etc. – see [3] and the references therein.

There already exist many algorithms for resolving this problem. All of them are based on alternating or cyclic projection onto each set E_i. Von Neumann [10] studied the special case $n = 2$, where each E_i is affine subspace, and Halperin [8] analyzed the case $n > 2$. See [4] for more exhaustive results on the affine case. The non-affine case was considered in [6,9]. By reinterpreting the former cyclic projection methods in a suitable Cartesian product space one can obtain iterative simultaneous projection methods [11].

In all existing methods the convergence rate is linear, and they can be presented by the following scheme: