Linear Algebra and its Applications 504 (2016) 468–486

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Two invariants for weak exponential stability of linear time-varying differential behaviors

Ulrich Oberst [1]

Institut für Mathematik, Universität Innsbruck, Technikerstrasse 13, A-6020 Innsbruck, Austria

ARTICLE INFO

Article history:
Received 16 February 2016
Accepted 6 April 2016
Available online 20 April 2016
Submitted by H. Gluesing-Luerssen

MSC:
93D20
93C15
93B25
34D20

Keywords:
Linear time-varying differential operator
Newton polygon
Graded module
Behavior
Weak exponential stability

ABSTRACT

In the paper [H. Bourlès, B. Marinescu, U. Oberst, Weak exponential stability (w.e.s.) of linear time-varying (LTV) differential behaviors, Linear Algebra Appl. 486 (2015) 523–571] we studied the problem of the title. If a finitely generated torsion module over an appropriate ring of differential operators and its associated autonomous system are regular singular the system is never w.e.s. In contrast we computed a square complex matrix for each irregular singular module and showed that the system is w.e.s. resp. not stable if all eigenvalues of the matrix have positive real parts resp. if at least one eigenvalue has negative real part. In this supplement of the quoted paper we show that the spectrum of the matrix and the decay exponent are isomorphy invariants of the module. The proofs make essential use of results exposed in [P. Maisonobe, C. Sabbah, $\mathcal{D}$-module cohérents et holonomes, Hermann, Paris, 1993]. We also complement the main w.e.s. result of our quoted paper by the case where at least one eigenvalue of the matrix is purely imaginary.

DOI of original article: http://dx.doi.org/10.1016/j.laa.2015.08.034.
E-mail address: ulrich.oberst@uibk.ac.at.

[1] Tel.: + 43 5238 53139.

http://dx.doi.org/10.1016/j.laa.2016.04.009
0024-3795/

1. Introduction

In [3] we studied the *weak exponential stability* (w.e.s.) of *linear time-varying differential* (LTV) systems [3, Def. 2.4], the varying coefficients being locally convergent *Puiseux series*. Every finitely generated left torsion module M over the appropriate integral domain of differential operators is interpreted as a system module and gives rise to a dual autonomous behavior $\mathcal{B}$. For an *irregular singular* module M we constructed a behavior isomorphism [3, Thm. 2.8]

$$\begin{aligned} &\mathcal{B} \cong \left\{x \in W(\tau)^n;\ \forall t > \tau:\ x'(t) + t^{\lambda-1}(A_0 + t^{-\mu}A_1(t^{-\mu}))x(t) = 0\right\} \\ &\text{where } 0 < \lambda, \mu \in \mathbb{Q},\ \mu^{-1} \in \mathbb{N},\ \mathbb{Z}\lambda \subseteq \mathbb{Z}\mu,\ 0 \neq A_0 \in \mathbb{C}^{n\times n},\ A_1 \in \mathbb{C} < z >^{n\times n} \end{aligned} \tag{1}$$

for sufficiently large $\tau \geq 0$; see (69)–(71) for slight differences of (1) to its original in [3]. The ring $\mathbb{C} < z >$ is the local domain of locally convergent power series. The signal space $W(\tau) := \mathrm{C}^\infty(\tau, \infty)$ consists of all smooth, complex-valued functions on the open interval $(\tau, \infty) := \{t \in \mathbb{R};\ t > \tau\}$. The w.e.s. of $\mathcal{B}$, i.e., the exponential decay of its trajectories for $t \to \infty$, is determined by the spectral properties of A_0 [3, Thm. 2.8]. The system is w.e.s. with decay factors $\exp(-\alpha t^\lambda)$ resp. not stable if all eigenvalues of A_0 have positive real part resp. at least one eigenvalue has negative real part. The proof of (1) made essential use of important results exposed in the excellent books [8] and [10].

In this supplement to [3] we show in Theorem 6.1 and in Corollary 7.2 that the number λ and the spectrum of A_0 are *isomorphy invariants* of M. Indeed λ is the largest positive slope of the *Newton polygon* of an associated differential operator and $\mathrm{spec}(A_0)$ is determined by the roots of its λ-*symbol*. Theorem 8.2 shows that the system is not w.e.s. with decay factors $\exp(-\alpha t^\lambda)$ if all eigenvalues of A_0 have nonnegative real part and at least one is purely imaginary. This is fully analogous to the case of constant coefficients. The system may, however, be w.e.s. with a decay factor $\exp(-\alpha' t^{\lambda'})$, $0 < \lambda' < \lambda$, $0 < \alpha'$, as Example 8.3 demonstrates. We prove the invariance of λ by extending the important invariance result [11, Cor. 1.6.11, p. 54], [8, Prop. I.5.1.4, Def. I.5.1.5] from Laurent series to Puiseux series. Up to an addition of 1 λ is also called the *irregularity* [7, p. 15] or *nonregularity* of M [2, p. 78]. We *simplify* the proof in [8] by avoiding the *good filtrations* [8, Ex. 5.13] of differential modules and their algebraic properties and *correct* two nontrivial errors of [3], see Remarks 4.2 and 6.2 for the precise statements. Our proof gives all details whereas that of [8] gives indications only that are hard to complete for systems theorists. We emphasize, however, that the essential ideas for the generalization, in particular the use of the λ-degree and the associated graded modules, come from [8]. As far as we see [10] does not contain the result from [8, Prop. 5.1.4]. We refer to the bibliography of [3] for important references concerning exponential stability of differential systems, for instance [13,6,5,1], and to [11,8,10,2] for the long and extensive history of the algebraic theory of linear differential systems with varying coefficients. The results of [3] and of the present paper are constructive, cf., for instance, [4,12], [10, Ch. 4], and