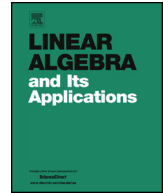




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## Perturbing eigenvalues of nonnegative matrices



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### ABSTRACT

Let  $A$  be an irreducible (entrywise) nonnegative  $n \times n$  matrix with eigenvalues

$$\rho, \lambda_2 = b + ic, \lambda_3 = b - ic, \lambda_4, \dots, \lambda_n,$$

where  $\rho$  is the Perron eigenvalue. It is shown that for any  $t \in [0, \infty)$  there is a nonnegative matrix with eigenvalues

$$\rho + \tilde{t}, \lambda_2 + t, \lambda_3 + t, \lambda_4, \dots, \lambda_n,$$

whenever  $\tilde{t} \geq \gamma_n t$  with  $\gamma_3 = 1$ ,  $\gamma_4 = 2$ ,  $\gamma_5 = \sqrt{5}$  and  $\gamma_n = 2.25$  for  $n \geq 6$ . The result improves that of Guo et al. Our proof depends on an auxiliary result in geometry asserting that the area of an  $n$ -sided convex polygon is bounded by  $\gamma_n$  times the maximum area of a triangle lying inside the polygon.

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## 1. Introduction

The *nonnegative inverse eigenvalue problem* concerns the study of necessary and sufficient conditions for a given set of complex numbers  $\lambda_1, \dots, \lambda_n$  to be the eigenvalues of an (entrywise) nonnegative matrix. This problem has attracted the attention of many authors, and is still open; for example, see [4,1,11] and their references. In connection to this study, researchers study the change of the Perron eigenvalue under the perturbation of the other real or complex eigenvalues of a given nonnegative matrix. Here are several results in this direction.

- (1) In [6], the author proved the following:

*Suppose  $\rho, \lambda_2, \lambda_3, \dots, \lambda_n$  are the eigenvalues of an  $n \times n$  nonnegative matrix  $A$  such that  $\rho$  is the Perron eigenvalue, and  $\lambda_2$  is real. Then, for any  $0 \leq t \leq \tilde{t}$ , there is a nonnegative matrix with eigenvalues  $\rho + \tilde{t}, \lambda_2 \pm t, \lambda_3, \dots, \lambda_n$ .*

- (2) Laffey [9] and Guo et al. [5] obtained the following independently:

*Suppose  $\rho, \lambda_2, \lambda_3, \dots, \lambda_n$  are the eigenvalues of an  $n \times n$  nonnegative matrix  $A$  such that  $\rho$  is the Perron eigenvalue, and  $(\lambda_2, \lambda_3) = (b + ic, b - ic)$  is a (non-real) complex conjugate pair. Then, for any  $\tilde{t}, t \in [0, \infty)$  with  $2t \leq \tilde{t}$ , there is a nonnegative matrix with eigenvalues  $\rho + \tilde{t}, \lambda_2 - t, \lambda_3 - t, \lambda_4, \dots, \lambda_n$ .*

- (3) In [5, Proposition 3.1], Guo and Guo showed that:

*Suppose  $\rho, \lambda_2, \lambda_3, \dots, \lambda_n$  are the eigenvalues of an  $n \times n$  nonnegative matrix  $A$  such that  $\rho$  is the Perron eigenvalue, and  $(\lambda_2, \lambda_3) = (b + ic, b - ic)$  is a (non-real) complex conjugate pair. Then, for any  $\tilde{t}, t \in [0, \infty)$  with  $4t \leq \tilde{t}$ , there is a nonnegative matrix with eigenvalues  $\rho + \tilde{t}, \lambda_2 + t, \lambda_3 + t, \lambda_4, \dots, \lambda_n$ .*

The authors also pose the problem of finding the smallest constant  $c$  for which the above result holds with  $4t$  replaced by  $ct$ . In [3] Cronin and Laffey show that  $c = 1$  for  $n = 3$ ,  $c = 2$  for  $n = 4$  and  $c \geq 2$  for  $n \geq 5$ . They further show that for  $c > 2$ , the result holds for sufficiently small  $t$  but the question about arbitrary  $t$  is left open.

The results in (1) and (2) above were shown to be optimal in the sense that the conclusion may fail if  $\tilde{t} < t$  in (1) and  $\tilde{t} < 2t$  in (2). However, the result in (3) may be strengthened. In this paper, we improve the third result in the following.

**Theorem 1.1.** *Suppose  $\rho, \lambda_2, \lambda_3, \dots, \lambda_n$  are the eigenvalues of an  $n \times n$  nonnegative matrix  $A$  such that  $\rho$  is the Perron eigenvalue, and  $\lambda_2 = b + ic$  and  $\lambda_3 = b - ic$  are (non-real) complex conjugate pairs. Then for any  $t \in [0, \infty)$  there is a nonnegative matrix with eigenvalues*

$$\rho + \tilde{t}, \lambda_2 + t, \lambda_3 + t, \lambda_4, \dots, \lambda_n,$$

*whenever  $\tilde{t} \geq \gamma_n t$  with  $\gamma_3 = 1$ ,  $\gamma_4 = 2$ ,  $\gamma_5 = \sqrt{5}$  and  $\gamma_n = 2.25$  for  $n \geq 6$ .*

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