Linear Algebra and its Applications 498 (2016) 3–20

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Perturbing eigenvalues of nonnegative matrices

Xuefeng Wang [a], Chi-Kwong Li [b], Yiu-Tung Poon [c,*]

[a] Department of Mathematics, Ocean University of China, Qingdao, Shandong 266100, China
[b] Department of Mathematics, College of William and Mary, Williamsburg, VA 23187, USA
[c] Department of Mathematics, Iowa State University, Ames, IA 50011, USA

ARTICLE INFO

Article history:
Received 3 February 2014
Accepted 8 April 2014
Available online 29 April 2014
Submitted by R. Brualdi

Dedicated to Professor Hans Schneider

MSC:
15A48
15A18

Keywords:
Nonnegative matrices
Perron eigenvalue
Perturbation

ABSTRACT

Let A be an irreducible (entrywise) nonnegative $n \times n$ matrix with eigenvalues

$$\rho, \lambda_2 = b + ic, \lambda_3 = b - ic, \lambda_4, \cdots, \lambda_n,$$

where ρ is the Perron eigenvalue. It is shown that for any $t \in [0, \infty)$ there is a nonnegative matrix with eigenvalues

$$\rho + \tilde{t}, \lambda_2 + t, \lambda_3 + t, \lambda_4, \cdots, \lambda_n,$$

whenever $\tilde{t} \geqslant \gamma_n t$ with $\gamma_3 = 1$, $\gamma_4 = 2$, $\gamma_5 = \sqrt{5}$ and $\gamma_n = 2.25$ for $n \geqslant 6$. The result improves that of Guo et al. Our proof depends on an auxiliary result in geometry asserting that the area of an n-sided convex polygon is bounded by γ_n times the maximum area of a triangle lying inside the polygon.
© 2014 Elsevier Inc. All rights reserved.

* Corresponding author.
E-mail addresses: wangxuefeng@ouc.edu.cn (X. Wang), ckli@math.wm.edu (C.-K. Li), ytpoon@iastate.edu (Y.-T. Poon).

http://dx.doi.org/10.1016/j.laa.2014.04.012
0024-3795/© 2014 Elsevier Inc. All rights reserved.

1. Introduction

The *nonnegative inverse eigenvalue problem* concerns the study of necessary and sufficient conditions for a given set of complex numbers $\lambda_1, \ldots, \lambda_n$ to be the eigenvalues of an (entrywise) nonnegative matrix. This problem has attracted the attention of many authors, and is still open; for example, see [4,1,11] and their references. In connection to this study, researchers study the change of the Perron eigenvalue under the perturbation of the other real or complex eigenvalues of a given nonnegative matrix. Here are several results in this direction.

(1) In [6], the author proved the following:
Suppose $\rho, \lambda_2, \lambda_3, \cdots, \lambda_n$ are the eigenvalues of an $n \times n$ nonnegative matrix A such that ρ is the Perron eigenvalue, and λ_2 is real. Then, for any $0 \leqslant t \leqslant \tilde{t}$, there is a nonnegative matrix with eigenvalues $\rho + \tilde{t}, \lambda_2 \pm t, \lambda_3, \cdots, \lambda_n$.

(2) Laffey [9] and Guo et al. [5] obtained the following independently:
Suppose $\rho, \lambda_2, \lambda_3, \cdots, \lambda_n$ are the eigenvalues of an $n \times n$ nonnegative matrix A such that ρ is the Perron eigenvalue, and $(\lambda_2, \lambda_3) = (b + ic, b - ic)$ is a (non-real) complex conjugate pair. Then, for any $\tilde{t}, t \in [0, \infty)$ with $2t \leqslant \tilde{t}$, there is a nonnegative matrix with eigenvalues $\rho + \tilde{t}, \lambda_2 - t, \lambda_3 - t, \lambda_4, \cdots, \lambda_n$.

(3) In [5, Proposition 3.1], Guo and Guo showed that:
Suppose $\rho, \lambda_2, \lambda_3, \cdots, \lambda_n$ are the eigenvalues of an $n \times n$ nonnegative matrix A such that ρ is the Perron eigenvalue, and $(\lambda_2, \lambda_3) = (b + ic, b - ic)$ is a (non-real) complex conjugate pair. Then, for any $\tilde{t}, t \in [0, \infty)$ with $4t \leqslant \tilde{t}$, there is a nonnegative matrix with eigenvalues $\rho + \tilde{t}, \lambda_2 + t, \lambda_3 + t, \lambda_4, \cdots, \lambda_n$.
The authors also pose the problem of finding the smallest constant c for which the above result holds with $4t$ replaced by ct. In [3] Cronin and Laffey show that $c = 1$ for $n = 3$, $c = 2$ for $n = 4$ and $c \geqslant 2$ for $n \geqslant 5$. They further show that for $c > 2$, the result holds for sufficiently small t but the question about arbitrary t is left open.

The results in (1) and (2) above were shown to be optimal in the sense that the conclusion may fail if $\tilde{t} < t$ in (1) and $\tilde{t} < 2t$ in (2). However, the result in (3) may be strengthened. In this paper, we improve the third result in the following.

Theorem 1.1. *Suppose $\rho, \lambda_2, \lambda_3, \cdots, \lambda_n$ are the eigenvalues of an $n \times n$ nonnegative matrix A such that ρ is the Perron eigenvalue, and $\lambda_2 = b + ic$ and $\lambda_3 = b - ic$ are (non-real) complex conjugate pairs. Then for any $t \in [0, \infty)$ there is a nonnegative matrix with eigenvalues*

$$\rho + \tilde{t}, \lambda_2 + t, \lambda_3 + t, \lambda_4, \cdots, \lambda_n,$$

whenever $\tilde{t} \geqslant \gamma_n t$ with $\gamma_3 = 1$, $\gamma_4 = 2$, $\gamma_5 = \sqrt{5}$ and $\gamma_n = 2.25$ for $n \geqslant 6$.

Download English Version:

https://daneshyari.com/en/article/6416063

Download Persian Version:

https://daneshyari.com/article/6416063

Daneshyari.com