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Jordan chains of h-cyclic matrices

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ABSTRACT

Arising from the classification of the matrix-roots of a nonnegative imprimitive irreducible matrix, we present results concerning the Jordan chains of an h-cyclic matrix. We also present ancillary results applicable to nonnegative imprimitive irreducible matrices and demonstrate these results via examples.

1. Introduction

The study of nonnegative matrices has its roots in the Perron–Frobenius Theorem, which asserts that a nonnegative irreducible matrix has a positive eigenvector associated

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with its spectral radius. This study has been extended to include results on reducible nonnegative matrices; for surveys of some of these results see [1,6,16]. Spectral properties of nonnegative matrices have proven very useful in the study of other related classes of matrices, such as M-matrices (see [1]) and eventually nonnegative matrices (see, e.g., [3–5,7,9,10,13,14,12,19,18]).

The focus of this article is to exploit the cyclicity of the spectrum of h-cyclic matrices to glean more information about the generalized eigenvectors of nonnegative and eventually nonnegative matrices. In [17], Tam observed that, for nonnegative irreducible imprimitive matrices, it is possible to predict the structure of the peripheral eigenvectors from the Perron vector. This idea is now extended to the generalized eigenvectors and Jordan chains of all the eigenvalues associated with any h-cyclic matrix, and irreducible nonnegative matrices in particular. We use these results in our paper on matrix-roots of imprimitive irreducible nonnegative matrices [11].

2. Notation and definitions

Denote by i the imaginary unit, i.e., $\mathrm{i} := \sqrt{-1}$. When convenient, an indexed set of the form $\{x_i, x_{i+1}, \ldots, x_{i+j}\}$ is abbreviated to $\{x_k\}_{k=i}^{i+j}$.

For $h \in \mathbb{N}$, $h > 1$,

$$R(h) := \{0, 1, \ldots, h-1\}$$

$$\omega := \exp(2\pi\mathrm{i}/h) \in \mathbb{C},$$

$$\Omega_h := \left\{\omega^k\right\}_{k=0}^{h-1} \subseteq \mathbb{C}, \tag{2.1}$$

and

$$\nu_h := \left(1, \omega, \ldots, \omega^{h-1}\right) \in \mathbb{C}^n. \tag{2.2}$$

Denote by $M_n(\mathbb{C})$ ($M_n(\mathbb{R})$) the algebra of complex (respectively, real) $n \times n$ matrices. Given $A \in M_n(\mathbb{C})$, the *spectrum* of A is denoted by $\sigma(A)$; the *spectral radius* of A is denoted by $\rho = \rho(A)$; and the *peripheral spectrum*, denoted by $\pi(A)$, is the multi-set given by

$$\pi(A) = \{\lambda \in \sigma(A) : |\lambda| = \rho\}.$$

The (block) (i,j)-entry of A is denoted by a_{ij} or $[A]_{ij}$ and the (block) entries of A are denoted by $[a_{ij}]$ or $[a_{ij}]_n^{i,j=1}$.

The *direct sum* of the matrices $A_1, \ldots, A_k$, where $A_i \in M_{n_i}(\mathbb{C})$, denoted by $A_1 \oplus \cdots \oplus A_k$, $\bigoplus_{i=1}^k A_i$, or $\operatorname{diag}(A_1, \ldots, A_k)$, is the $n \times n$ matrix