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Characterization of a family of generalized companion matrices

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Matrices A of order n having entries in the field $\mathbb{F}(x_1, \ldots, x_n)$ of rational functions over a field $\mathbb{F}$ and characteristic polynomial

$$\det(tI - A) = t^n + x_1 t^{n-1} + \cdots + x_{n-1} t + x_n$$

are studied. It is known that such matrices are irreducible and have at least $2n - 1$ nonzero entries. Such matrices with exactly $2n - 1$ nonzero entries are called Ma–Zhan matrices. Conditions are given that imply that a Ma–Zhan matrix is similar via a monomial matrix to a generalized companion matrix (that is, a lower Hessenberg matrix with ones on its superdiagonal, and exactly one nonzero entry in each of its subdiagonals). Via the Ax–Grothendieck Theorem (respectively, its analog for the reals) these conditions are shown to hold for a family of matrices whose entries are complex (respectively, real) polynomials.

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1. Families of companion-like matrices

The $n \times n$ *Frobenius companion matrix* is the matrix

$$C_n = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -x_n & -x_{n-1} & -x_{n-2} & \cdots & -x_1 \end{bmatrix} \tag{1}$$

and its characteristic polynomial, $\det(tI - C_n)$, is

$$t^n + x_1 t^{n-1} + x_2 t^{n-2} + \cdots + x_{n-1} t + x_n. \tag{2}$$

Recently several generalizations of the Frobenius companion matrix have been studied [3,4,7]. In [7], Ma and Zhan investigate $n \times n$ matrices whose entries are in the field $\mathbb{F}(x_1, x_2, \ldots, x_n)$ of rational functions in the variables $x_1, x_2, \ldots, x_n$ for which the characteristic polynomial is (2), where $\mathbb{F}$ is a given field. They show that such a matrix is necessarily irreducible and has at least $2n-1$ nonzero entries. We define a *Ma–Zhan matrix* to be an $n \times n$ matrix over $\mathbb{F}(x_1, x_2, \ldots, x_n)$ with exactly $2n-1$ nonzero entries and characteristic polynomial (2). The results of [7] lead to the motivation for the problem addressed here, namely to characterize the $n \times n$ Ma–Zhan matrices.

There are many examples of $n \times n$ Ma–Zhan matrices. Indeed, Fiedler [4] gives a family of such matrices and more recently Eastman et al. [3] present two richer families. These families all have a common form, which we now describe. Each matrix A in the families has all zeros above the superdiagonal, all ones on the superdiagonal, and for $i = 0, \ldots, n-1$ contains exactly one nonzero entry, $-y_{i+1}$, on its i-th subdiagonal (that is, the positions $(i+1,1), (i+2,2), \ldots, (n, n-i)$). Implicit in [3], for each such A there exist unique $p_1, p_2, \ldots, p_n$ in the ring of polynomials $\mathbb{F}[x_1, \ldots, x_n]$ in $x_1, \ldots, x_n$ such that if $y_i = p_i$ $(i = 1, \ldots, n)$, then the characteristic polynomial of A is (2). For example, if

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -y_2 & 0 & 1 & 0 \\ -y_3 & 0 & -y_1 & 1 \\ -y_4 & 0 & 0 & 0 \end{bmatrix},$$

then the characteristic polynomial of A is $t^4 + y_1 t^3 + y_2 t^2 + (y_3 + y_1 y_2)t + y_4$. Setting $y_1 = x_1$, $y_2 = x_2$, $y_3 = x_3 - y_1 y_2 = x_3 - x_1 x_2$ and $y_4 = x_4$ yields the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -x_2 & 0 & 1 & 0 \\ -x_3 + x_1 x_2 & 0 & -x_1 & 1 \\ -x_4 & 0 & 0 & 0 \end{bmatrix}, \tag{3}$$

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