Linear Algebra and its Applications 498 (2016) 490–520

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

On scaling to an integer matrix and graphs with integer weighted cycles

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ARTICLE INFO

Article history:
Received 11 December 2014
Accepted 12 January 2016
Available online 21 January 2016
Submitted by R. Brualdi

MSC:
15A24
15A15
15A80

Keywords:
Max-plus algebra
Diagonal similarity scaling
Direct similarity scaling
Symmetric scaling
Integer matrices
Integer cycles

ABSTRACT

Between 1970 and 1982 Hans Schneider and co-authors produced a number of results regarding matrix scalings. They demonstrated that a matrix has a diagonal similarity scaling to any matrix with entries in the subgroup generated by the cycle weights of the associated digraph, and that a matrix has an equivalent scaling to any matrix with entries related to the weights of cycles in an associated bipartite graph. Further, given matrices A and B, they produced a description of all diagonal X such that $X^{-1}AX = B$. In 2005 Butkovič and Schneider used max-algebra to give a simple and efficient description of this set of scalings. In this paper we focus on the additive group of integers, and work in the max-plus algebra to give a full description of all scalings of a real matrix A to any integer matrix. We do this for four types of scalings; beginning with the familiar $X^{-1}AX$, XAY and XAX scalings and finishing with a new scaling which we call a signed similarity scaling. This is a scaling of the form XAY where we specify for each row i, either $x_i = y_i$ or $x_i = -y_i$. In all of our results we use necessary and sufficient conditions for existence which are based on integer weighted cycles in

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http://dx.doi.org/10.1016/j.laa.2016.01.018
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the associated digraph, or associated bipartite graph, of the matrix.

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1. Introduction

In [5,6,8] the authors study matrix scalings of the form $X^{-1}AX$ and XAY. Given an irreducible matrix $A \in G^{n\times n}$ for some group G, they considered in [5] the problem of determining whether there exists a diagonal similarity scaling of A to a matrix with entries in a subgroup of G. Specifically they showed that if H is the subgroup of G generated by the weights of cycles in A, then there exists X such that $X^{-1}AX \in H^{n\times n}$. Further they described how to find one such X. They extend these results to scalings of the form XAY. In [8] they gave necessary and sufficient conditions for two matrices $A, B \in G^{m\times n}$ to be diagonally similar (for $m = n$) or directly similar (called diagonally equivalent in that paper). Additionally, for additive groups G, they demonstrated that diagonal similarity can be characterised in terms of flows on the corresponding graph of the matrices, and direct similarity in terms of flows on the bipartite graph. In [6], given A and B, they gave a full description of all X such that $X^{-1}AX = B$. The authors in [3] demonstrated that max-times algebra (which is isomorphic to the max-plus algebra) could be used to describe the set of all such scalings satisfying $XAX = B$ in a simple and efficient manner. Symmetric scalings, that is scalings of the form XAX, have also been extensively studied, often in relation to specified row and column sums, or row and column maxima, see e.g. [7].

In this paper we focus on the additive group of integers. We introduce a new scaling problem for square matrices which we call a signed similarity scaling. This is a scaling of the form XAY where we specify for each row i, whether $x_i = y_i$ or $x_i = -y_i$, thus it contains both $X^{-1}AX$ and XAX scalings as subproblems. We give necessary and sufficient conditions for a matrix to have a signed similarity scaling to an integer matrix, and additionally describe all possible scalings to an integer matrix for finite (real) matrices. We work in the max-plus algebra as it allows us a simple way of describing the set of all solutions to the problems we consider. In order to state our main results we first consider the problems of determining whether a matrix A has a diagonal similarity, direct similarity or symmetric scaling to an integer matrix. Sections 3 and 4 contain the results on diagonal similarity scalings and direct similarity scalings respectively. In these sections we state necessary and sufficient conditions for a matrix to have a scaling to an integer matrix. Although these conditions follow from the results in [5,8] we state them using simple terminology that ties together all the problems we consider. Additionally we are able to give a complete and simple description of all scalings of A to an integer matrix. In Section 6 we perform the same procedure for symmetric scalings. Finally in Section 7 we bring together all our previous results to give necessary and sufficient

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