

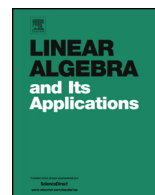


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# Linear Algebra and its Applications

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## Non-archimedean valuations of eigenvalues of matrix polynomials



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We dedicate this article to the memory of Hans Schneider

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### ABSTRACT

We establish general weak majorization inequalities, relating the leading exponents of the eigenvalues of matrices or matrix polynomials over the field of Puiseux series with the tropical analogues of eigenvalues. We also show that these inequalities become equalities under genericity conditions, and that the leading coefficients of the eigenvalues are determined as the eigenvalues of auxiliary matrix polynomials.

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## 1. Introduction

### 1.1. Non-archimedean valuations and tropical geometry

A non-archimedean valuation  $\nu$  on a field  $\mathbb{K}$  is a map  $\mathbb{K} \rightarrow \mathbb{R} \cup \{+\infty\}$  such that

$$\nu(a) = +\infty \iff a = 0 \quad (1a)$$

$$\nu(a + b) \geq \min(\nu(a), \nu(b)) \quad (1b)$$

$$\nu(ab) = \nu(a) + \nu(b) \quad (1c)$$

These properties imply that  $\nu(a+b) = \min(\nu(a), \nu(b))$  for  $a, b \in \mathbb{K}$  such that  $\nu(a) \neq \nu(b)$ . Therefore, the map  $\nu$  is almost a morphism from  $\mathbb{K}$  to the *min-plus* or *tropical* semifield  $\mathbb{R}_{\min}$ , which is the set  $\mathbb{R} \cup \{+\infty\}$ , equipped with the addition  $(a, b) \mapsto \min(a, b)$  and the multiplication  $(a, b) \mapsto a+b$ . A basic example of field with a non-archimedean valuation is the field of complex Puiseux series, with the valuation which takes the leading (smallest) exponent of a series. The images by a non-archimedean valuation of algebraic subsets of  $\mathbb{K}^n$  are known as *non-archimedean amoebas*. The latter have a combinatorial structure which is studied in tropical geometry [30,42]. For instance, Kapranov's theorem shows that the closure of the image by a non-archimedean valuation of an algebraic hypersurface over an algebraically closed field is a tropical hypersurface, i.e., the non-differentiability locus of a convex polyhedral function, see [23]. This generalizes the characterization of the leading exponents of the different branches of an algebraic curve in terms of the slopes of the Newton polygon, which is part of the classical Newton–Puiseux theorem.

### 1.2. Main results

In the present paper, we consider the eigenproblem over the field of complex Puiseux series and related fields of functions. Our aim is to relate the images of the eigenvalues by the non-archimedean valuation with certain easily computable combinatorial objects called *tropical eigenvalues*.

The first main result of the present paper, [Theorem 4.4](#), shows that the sequence of valuations of the eigenvalues of a matrix  $\mathcal{A} \in \mathbb{K}^{n \times n}$  is weakly (super) majorized by the sequence of (algebraic) tropical eigenvalues of the matrix obtained by applying the valuation to the entries of  $\mathcal{A}$ . Next, we show that the same majorization inequality holds under more general circumstances. In particular, in [Theorem 5.2](#) and [Corollary 5.3](#), we deal with a relaxed definition of the valuation. In the spirit of large deviations theory, we assume that the entries of the matrix are functions of a small parameter  $\epsilon$ , which may not have a Puiseux series type expansion, but have some mild form of first order

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