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Two-step modulus-based matrix splitting iteration method for a class of nonlinear complementarity problems ☆

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ABSTRACT

In this paper, we reformulate the nonlinear complementarity problem as an implicit fixed-point equation. We establish a modulus-based matrix splitting iteration method based on the implicit fixed-point equation and prove its convergence theorem under suitable conditions. Furthermore, we propose a two-step modulus-based matrix splitting iteration method, which may achieve higher computing efficiency. We can obtain many matrix splitting iteration methods by suitably choosing the matrix splittings and the parameters. The proposed methods can be regarded as extensions of the methods for linear complementarity problem. Numerical experiments are presented to show the effectiveness of the proposed methods.

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1. Introduction

In this paper, we consider the following nonlinear complementarity problem (NCP): finding a $u \in R^n$ such that

$$u \geq 0, \quad v = F(u) \geq 0, \quad u^T v = 0, \tag{1.1}$$

where the function F has the form of $Au + \Phi(u) + q$, $A = (a_{ij}) \in R^{n\times n}$ is a given matrix, q is a given vector, $\Phi : R^n \to R^n$ is a given diagonal differentiable mapping, that is, the ith component Φ_i of Φ is a function of the ith variable u_i only:

$$\Phi_i = \Phi_i(u_i), \qquad i = 1, 2, \ldots, n.$$

Obviously, if Φ is a linear function, problem (1.1) reduces to a linear complementarity problem.

We call (1.1) an nonlinear complementarity problem with a nonlinear source term. Problem (1.1) has many applications, especially in engineering, see for example [1–3]. There are many ways to compute a numerical solution of the NCP, such as linearized projected relaxation methods [4], nonlinear multisplitting iteration methods [5–7], multigrid methods [8] and domain decomposition methods [9,10]. Most of these methods need to solve linear complementarity subproblems, see [11–14] for several typical iteration methods. Besides, inexact semismooth Newton methods have been developed to solve problem (1.1) based on its semismooth reformulation [15,16]. This method is attractive because it converges rapidly from any sufficiently good initial iterate and the subproblems are systems of equations.

Recently, a new method called modulus-based matrix splitting iteration method for linear complementarity problem (LCP) attracts a great deal of attention. By reformulating the LCP as an implicit fixed-point equation, Bai [17] presented modulus-based matrix splitting iteration methods for LCP. When the system matrix A is an H_+-matrix [13], Xu and Liu [18] improved convergence theories of modulus-based matrix splitting iteration methods. Li [19] extended the modulus-based matrix splitting iteration method to general cases. Bai and Zhang [20] constructed modulus-based synchronous two-stage multisplitting iteration methods based on two-stage multisplitting of the system matrices. These iteration methods include the multisplitting relaxation methods as special cases. Applying the nonsmooth Newton's method to the equivalent reformulation of the LCP, Zheng and Li [21] presented a modulus-based nonsmooth Newton's method and established its locally quadratical convergence theory. Recently, Xia and Li [22] presented some modulus-based matrix splitting iteration methods for NCP. They established the convergence theories for certain system matrices. Motivated by recent work in this field, we will present a two-step modulus-based matrix splitting iteration method for NCP and discuss its convergence.

The remainder of this paper is structured as follows: Section 2 presents the modulus-based matrix splitting iteration methods and discusses their convergence. Section 3