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Spectral condition for Hamiltonicity of a graph

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ABSTRACT

In this paper, we give a sufficient condition on the spectral radius for a graph to be Hamiltonian.

1. Introduction

Throughout this paper all graphs are finite, simple and undirected. Let $G = (V(G), E(G))$ be a graph of order $n = |V(G)|$ and of size $m = |E(G)|$ and let $A = A(G)$ be its adjacency matrix. Since A is symmetric, its eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ are real. We assume that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$ and refer to $\lambda_1, \lambda_2, \ldots, \lambda_n$ as the spectrum of G. Besides, for each eigenvalue λ_i of A its algebraic and geometric multiplicities are equal (the root subspaces coincide with the eigensubspaces). Since A is nonnegative ($a_{ij} \geq 0$ for

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any element of A), then λ_1 is nonnegative and has the maximum absolute value among all eigenvalues of A. So we can give the following definition.

Definition 1. The largest eigenvalue λ_1 is called the spectral radius of G, denoted by $\rho(G)$.

According to the Perron–Frobenius theory of matrices [1] for the spectral radius $\rho(G)$ there exists a nonnegative eigenvector corresponding to $\rho(G)$.

Let $N_G(v) = N(v)$ denote the neighbour set of a vertex v in G, and let $N_G[v] = N[v] = N(v) \cup \{v\}$. The degree of any vertex $v_i \in V$ of the graph G, denoted by $d_i = \deg_G(v_i)$, is equal to $|N(v_i)|$. Let $(d_1, d_2, \ldots, d_n)$ be the degree sequence of the graph G, where $d_1 \leq d_2 \leq \ldots \leq d_n$. Then $d_1 = \delta$ is called *the minimum degree.*

Definition 2. *The union* of two graphs G and H is the graph $G \cup H$ with the vertex set $V(G) \cup V(H)$ and the edge set $E(G) \cup E(H)$. If graphs G and H are disjoint, then we call their union *a disjoint union* and denote it by $G + H$. The union of k disjoint copies of a graph G is denoted by kG. *The join* of two disjoint graphs G and H, denoted by $G \vee H$, is obtained from $G + H$ by joining each vertex of G to each vertex of H.

Denote by K_n a complete graph of order n and by $K_{m,n}$ a complete bipartite graph with vertex classes of sizes m and n.

Definition 3. A cycle or path passing through all the vertices of a graph is called *Hamiltonian.* A graph G, containing a Hamiltonian cycle or path, is called *Hamiltonian* or *traceable* correspondingly.

It is known that the problem of deciding whether a given graph is Hamiltonian or traceable is NP-complete. Many sufficient or necessary conditions were given for a graph to be Hamiltonian or traceable. Furthermore, spectral graph theory has been applied to this problem.

In [2] Brualdi and Solheid raised the following spectral problem.

Problem 1. What is the maximum spectral radius of a graph G on n vertices belonging to a specified class of graphs?

Recently, the following important Brualdi–Solheid–Turan type problem has been intensively studied:

Problem 2. For a given graph F, what is the maximum spectral radius of a graph G on n vertices without a subgraph isomorphic to F?

Up till now, Problem 2 has been considered for many cases, in particular, where F is a clique, an even or odd path (cycle) of the given length and a Hamiltonian path (cycle) [3–9].