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Lower bounds of Nikiforov's energy over digraphs

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A B S T R A C T

The energy of a graph G is defined as $E(G) = \sum_{i=1}^{n} |\lambda_i|$, where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of the adjacency matrix of G. This concept was extended by Nikiforov [8] to digraphs as $\mathcal{N}(D) = \sum_{i=1}^{n} \sigma_i$, where D is a digraph with n vertices and singular values $\sigma_1, \ldots, \sigma_n$. Upper bounds of $\mathcal{N}$ were found by Kharaghani and Tayfeh-Rezaie [4]. In this work we find lower bounds of $\mathcal{N}$ over the set of digraphs.

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1. Introduction

Let D be a digraph without loops and multiple edges. Assume that the set of vertices is given by $\{1, 2, \ldots, n\}$. We write ij to indicate that there exists an arc from vertex i to vertex j. A digraph D is said to be symmetric if when ij is an arc, then also ji is an arc. In this case we say that ij and ji is a pair of symmetric arcs of D. A graph G can be viewed as a symmetric digraph, where every edge is identified with a pair of symmetric arcs. On the other hand a digraph D is asymmetric when D has no pair of symmetric arcs.

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The adjacency matrix of a digraph D with n vertices is defined as the $n \times n$ matrix A whose element ij is

$$[A]_{ij} = \begin{cases} 1 & \text{if } ij \text{ is an arc} \\ 0 & \text{if not}\,. \end{cases}$$

The characteristic polynomial of D, denoted by $\phi_D(x)$, is defined as the characteristic polynomial of the matrix A, i.e., $\phi_D(x) = det(x\,I - A)$, where I is the $n \times n$ identity. The eigenvalues of D are the eigenvalues of A and the singular values of D are the singular values of A.

If G is a graph then the adjacency matrix is symmetric and so the eigenvalues $\lambda_1, \ldots, \lambda_n$ are real numbers. The energy of a graph G is denoted by $E(G)$ and defined as $E(G) = \sum_{i=1}^{n} |\lambda_i|$. This concept was introduced for general graphs by I. Gutman [3] and it is related to the total π-electron energy in a molecule represented by a (molecular) graph. For the mathematical properties of the energy of a graph we refer to the book [6]. This concept has been extended in several ways to digraphs. Specifically in this paper we consider the digraph energies $\mathcal{S}(D) = \sum_{i=1}^{n} |z_i|$ [7], $\mathcal{E}(D) = \sum_{i=1}^{n} |\mathrm{Re}(z_i)|$ [9] and Nikiforov's energy $\mathcal{N}(D) = \sum_{i=1}^{n} \sigma_i$ [8], where D is a digraph with n vertices, eigenvalues $z_1, \ldots, z_n$ and singular values $\sigma_1, \ldots, \sigma_n$.

Koolen–Moulton-type upper bounds [5] of $\mathcal{N}$ were considered for general matrices and $(0,1)$-matrices in the papers [4] and [8]. In particular for digraphs, one obtains upper bounds for Nikiforov's energy in terms of the number of vertices and number of arcs. More precisely, recall that the $n \times n$ matrix A is the incidence matrix of a symmetric (n, k, λ)-BIBD if and only if

$$AA^{\top} = \lambda J + (k - \lambda)\, I \tag{1}$$

where J is the all 1 matrix and I is the identity matrix. If the adjacency matrix A of a digraph D satisfies (1) then we say that D is a symmetric (n, k, λ)-BIBD.

Theorem 1.1. *Let D be a digraph with n vertices and a arcs. If $a \geq n$ then*

1. $\mathcal{N}(D) \leq \frac{a}{n} + \sqrt{(n-1)\left(a - \left(\frac{a}{n}\right)^2\right)}$. *Equality occurs if and only if D is a symmetric $\left(n, \frac{a}{n}, \frac{a(a-n)}{n^2(n-1)}\right)$-BIBD;*
2. $\mathcal{N}(D) \leq \frac{n(1+\sqrt{n})}{2}$. *Equality occurs if and only if D is a symmetric $\left(n, \frac{n+\sqrt{n}}{2}, \frac{n+2\sqrt{n}}{4}\right)$-BIBD.*

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