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Classification of five-dimensional nilpotent Jordan algebras

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A B S T R A C T

The paper is devoted to classify nilpotent Jordan algebras of dimension up to five over an algebraically closed field $\mathbb{F}$ of characteristic not 2. We obtained a list of 35 isolated non-isomorphic 5-dimensional nilpotent non-associative Jordan algebras and 6 families of non-isomorphic 5-dimensional nilpotent non-associative Jordan algebras depending either on one or two parameters over an algebraically closed field of characteristic $\neq 2, 3$. In addition to these algebras we obtained two non-isomorphic 5-dimensional nilpotent non-associative Jordan algebras over an algebraically closed field of characteristic 3.

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1. Introduction

The classification, up to isomorphism, of any class of algebras is a fundamental and very difficult problem. The classification of nilpotent Jordan algebras is one of the fundamental problems. In this way nilpotent Jordan algebras of small dimension were classified. Nilpotent associative Jordan algebras of dimension up to five over algebraically closed

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fields of characteristic not $2, 3$ were classified in [6]. Nilpotent associative Jordan algebras of dimension up to five over algebraically closed fields were classified in [7]. Nilpotent Jordan algebras of dimension up to four over $\mathbb{C}$ were classified in [1].

The aim of this paper is to complete the classification of 5-dimensional nilpotent Jordan algebras over an algebraically closed field of characteristic not 2. To this end, we classify 5-dimensional nilpotent non-associative Jordan algebras over an algebraically closed field of characteristic not 2.

In this paper we describe a method for classifying nilpotent Jordan algebras. Subsequently we classify, up to isomorphism, all nilpotent Jordan algebras of dimension up to four and all nilpotent non-associative Jordan algebras of dimension 5 over an algebraically closed field $\mathbb{F}$ of characteristic not 2. This method is analogous to the *Skjelbred–Sund* method for classifying nilpotent Lie algebras (see [2,3,8]). We introduce a new invariant called the *characteristic sequence of dimensions of radicals*. It offers us a very effective way to distinguish two algebras.

The paper is organized as follows. In Section 2 we introduce the notion of annihilator extension. In Sections 3, 4 we describe a method for classification of nilpotent Jordan algebras. In Section 5 the classification of nilpotent Jordan algebras up to dimension three is given. Section 6 contains a complete classification of 4-dimensional nilpotent Jordan algebras. Section 7 is devoted to classify 5-dimensional nilpotent non-associative Jordan algebras.

2. Annihilator extension of Jordan algebra

Definition 2.1. A *Jordan algebra* J is a commutative algebra over a field $\mathbb{F}$ with a multiplication "$\circ$" satisfying the *Jordan identity*

$$x^2 \circ (x \circ y) = x \circ (x^2 \circ y) \text{ for all } x, y \in J. \tag{2.1}$$

The linearization of the Jordan identity (2.1) is

$$(x, y, z \circ w) + (w, y, z \circ x) + (z, y, x \circ w) = 0 \tag{2.2}$$

for all $x, y, z, w \in J$. Here $(x, y, z) = (x \circ y) \circ z - x \circ (y \circ z)$ is the associator of x, y, z.

Definition 2.2. Let S be a subset of a Jordan algebra J. The set $Ann(S) = \{x \in J : x \circ S = 0\}$ is called the *annihilator* of S.

In a Jordan algebra J we define inductively a series of subsets by setting $J^{\langle 1\rangle} = J$ and $J^{\langle n\rangle} = J^{\langle n-1\rangle} \circ J$. The chain of subsets

$$J^{\langle 1\rangle} \supseteq J^{\langle 2\rangle} \supseteq \cdots \supseteq J^{\langle n\rangle} \supseteq \cdots$$

is a chain of ideals of the algebra J.

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