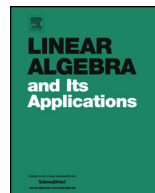




ELSEVIER

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa


Spectral conditions for edge connectivity and packing spanning trees in multigraphs



Xiaofeng Gu

Department of Mathematics, University of West Georgia, Carrollton, GA 30118,
USA

ARTICLE INFO

Article history:

Received 28 January 2015

Accepted 30 November 2015

Available online 17 December 2015

Submitted by R. Brualdi

MSC:

05C50

05C05

05C40

Keywords:

Eigenvalue

Algebraic connectivity

Edge connectivity

Spanning tree

Balloon

ABSTRACT

A multigraph is a graph with possible multiple edges, but no loops. The multiplicity of a multigraph is the maximum number of edges between any pair of vertices. We prove that, for a multigraph G with multiplicity m and minimum degree $\delta \geq 2k$, if the algebraic connectivity is greater than $\min\{\frac{2k-1}{\lceil(\delta+1)/m\rceil}, \frac{2k-1}{2}\}$, then G has at least k edge-disjoint spanning trees; for a multigraph G with multiplicity m and minimum degree $\delta \geq k$, if the algebraic connectivity is greater than $\min\{\frac{2(k-1)}{\lceil(\delta+1)/m\rceil}, k-1\}$, then the edge connectivity is at least k . These extend some earlier results.

A balloon of a graph G is a maximal 2-edge-connected subgraph that is joined to the rest of G by exactly one cut-edge. We provide spectral conditions for the number of balloons in a multigraph, which also generalizes an earlier result.

© 2015 Elsevier Inc. All rights reserved.

1. Introduction

A **multigraph** is a graph with possible multiple edges, but no loops. The **multiplicity** of a multigraph is the maximum number of edges between any pair of vertices. In

E-mail address: xgu@westga.edu.

<http://dx.doi.org/10.1016/j.laa.2015.11.038>

0024-3795/© 2015 Elsevier Inc. All rights reserved.

this paper, we consider finite undirected multigraphs. Thus, “a graph” in this paper means “a multigraph”, unless otherwise stated. We follow the notations of Bondy and Murty [1] for undefined terms. If X_1, X_2, \dots, X_t are disjoint vertex subsets of G , then $e(X_1, X_2, \dots, X_t)$ denotes the number of edges with two endpoints in distinct X_i 's, for $i = 1, 2, \dots, t$. In the following, we assume k and d are positive integers.

Let G be an undirected graph with vertex set $\{v_1, v_2, \dots, v_n\}$. The **adjacency matrix** of G is an n by n matrix $A(G)$ with entry a_{ij} being the number of edges between v_i and v_j for $1 \leq i, j \leq n$. We use $\lambda_i(G)$ to denote the i th largest eigenvalue of $A(G)$; when the graph G is understood from the context, we use λ_i for $\lambda_i(G)$. With these notations, we always have $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$. Let $D(G)$ be the **degree matrix** of G , that is, the n by n diagonal matrix with entry a_{ii} being the degree of vertex v_i in G for $1 \leq i \leq n$. The matrices $L(G) = D(G) - A(G)$ and $Q(G) = D(G) + A(G)$ are the **Laplacian matrix** and the **signless Laplacian matrix** of G , respectively. We use $\mu_i(G)$ and $q_i(G)$ to denote the i th largest eigenvalue of $L(G)$ and $Q(G)$, respectively. It is not difficult to see that $\mu_n(G) = 0$. The second smallest eigenvalue of $L(G)$, $\mu_{n-1}(G)$, is known as the **algebraic connectivity** of G .

For a graph G , the **spanning tree packing number**, denoted by $\tau(G)$, is the maximum number of edge-disjoint spanning trees in G . A fundamental theorem characterizing graphs G with $\tau(G) \geq k$ has been obtained by Nash-Williams and Tutte.

Theorem 1.1. (Nash-Williams [11] and Tutte [15]) *Let G be a connected graph. Then $\tau(G) \geq k$ if and only if for any partition (V_1, \dots, V_t) of $V(G)$, $e(V_1, \dots, V_t) \geq k(t - 1)$.*

Cioabă and Wong [4] investigated the relationship between the second largest adjacency eigenvalue and $\tau(G)$ for a regular simple graph G , and made **Conjecture 1.1**. Utilizing **Theorem 1.1**, Cioabă and Wong proved the conjecture for $k \in \{2, 3\}$.

Conjecture 1.1. (Cioabă and Wong [4]) *Let k and d be two integers with $d \geq 2k \geq 4$. If G is a d -regular simple graph with $\lambda_2(G) < d - \frac{2k-1}{d+1}$, then $\tau(G) \geq k$.*

Conjecture 1.1 was then extended to **Conjecture 1.2** for any simple graph G (not necessarily regular). See [5,6,8,9] for the conjecture and some partial results.

Conjecture 1.2. (Gu [5], Gu, Lai, Li and Yao [6], Li and Shi [8], Liu, Hong and Lai [9]) *Let k be an integer with $k \geq 2$ and G be a simple graph with minimum degree $\delta \geq 2k$. If $\lambda_2(G) < \delta - \frac{2k-1}{\delta+1}$, then $\tau(G) \geq k$.*

Recently, both **Conjectures 1.1 and 1.2** were settled in [10].

Theorem 1.2. (Liu, Hong, Gu and Lai [10]) *Let G be a simple graph with $\delta \geq 2k$.*

- (i) *If $\mu_{n-1}(G) > \frac{2k-1}{\delta+1}$, then $\tau(G) \geq k$.*
- (ii) *If $\lambda_2(G) < \delta - \frac{2k-1}{\delta+1}$, then $\tau(G) \geq k$.*
- (iii) *If $q_2(G) < 2\delta - \frac{2k-1}{\delta+1}$, then $\tau(G) \geq k$.*

Download English Version:

<https://daneshyari.com/en/article/6416157>

Download Persian Version:

<https://daneshyari.com/article/6416157>

[Daneshyari.com](https://daneshyari.com)