

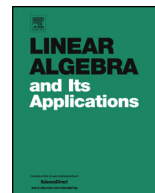


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# Concentration of the mixed discriminant of well-conditioned matrices



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## ARTICLE INFO

*Article history:*

Received 31 August 2015

Accepted 17 November 2015

Available online 17 December 2015

Submitted by R. Brualdi

*MSC:*

15A15

15A45

90C25

68Q25

*Keywords:*

Mixed discriminant

Scaling

Algorithm

Concentration

## ABSTRACT

We call an  $n$ -tuple  $Q_1, \dots, Q_n$  of positive definite  $n \times n$  real matrices  $\alpha$ -conditioned for some  $\alpha \geq 1$  if for the corresponding quadratic forms  $q_i : \mathbb{R}^n \rightarrow \mathbb{R}$  we have  $q_i(x) \leq \alpha q_i(y)$  for any two vectors  $x, y \in \mathbb{R}^n$  of Euclidean unit length and  $q_i(x) \leq \alpha q_j(x)$  for all  $1 \leq i, j \leq n$  and all  $x \in \mathbb{R}^n$ . An  $n$ -tuple is called doubly stochastic if the sum of  $Q_i$  is the identity matrix and the trace of each  $Q_i$  is 1. We prove that for any fixed  $\alpha \geq 1$  the mixed discriminant of an  $\alpha$ -conditioned doubly stochastic  $n$ -tuple is  $n^{O(1)} e^{-n}$ . As a corollary, for any  $\alpha \geq 1$  fixed in advance, we obtain a polynomial time algorithm approximating the mixed discriminant of an  $\alpha$ -conditioned  $n$ -tuple within a polynomial in  $n$  factor.

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## 1. Introduction and main results

### 1.1. Mixed discriminants

Let  $Q_1, \dots, Q_n$  be  $n \times n$  real symmetric matrices.

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<http://dx.doi.org/10.1016/j.laa.2015.11.040>

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The function  $\det(t_1Q_1 + \dots + t_nQ_n)$ , where  $t_1, \dots, t_n$  are real variables, is a homogeneous polynomial of degree  $n$  in  $t_1, \dots, t_n$  and its coefficient

$$D(Q_1, \dots, Q_n) = \frac{\partial^n}{\partial t_1 \dots \partial t_n} \det(t_1Q_1 + \dots + t_nQ_n) \tag{1.1.1}$$

is called the *mixed discriminant* of  $Q_1, \dots, Q_n$  (sometimes, the normalizing factor of  $1/n!$  is used). Mixed discriminants were introduced by A.D. Alexandrov in his work on mixed volumes [1], see also [8]. They also have some interesting combinatorial applications, see Chapter V of [3].

Mixed discriminants generalize permanents. If the matrices  $Q_1, \dots, Q_n$  are diagonal, so that

$$Q_i = \text{diag}(a_{i1}, \dots, a_{in}) \quad \text{for } i = 1, \dots, n,$$

then

$$D(Q_1, \dots, Q_n) = \text{per } A \quad \text{where } A = (a_{ij}) \tag{1.1.2}$$

and

$$\text{per } A = \sum_{\sigma \in S_n} \prod_{i=1}^n a_{i\sigma(i)}$$

is the *permanent* of an  $n \times n$  matrix  $A$ . Here the  $i$ -th row of  $A$  is the diagonal of  $Q_i$  and  $S_n$  is the symmetric group of all  $n!$  permutations of the set  $\{1, \dots, n\}$ .

### 1.2. Doubly stochastic $n$ -tuples

If  $Q_1, \dots, Q_n$  are positive semidefinite matrices then  $D(Q_1, \dots, Q_n) \geq 0$ , see [8]. We say that the  $n$ -tuple  $(Q_1, \dots, Q_n)$  is *doubly stochastic* if  $Q_1, \dots, Q_n$  are positive semidefinite,

$$Q_1 + \dots + Q_n = I \quad \text{and} \quad \text{tr } Q_1 = \dots = \text{tr } Q_n = 1,$$

where  $I$  is the  $n \times n$  identity matrix and  $\text{tr } Q$  is the trace of  $Q$ . We note that if  $Q_1, \dots, Q_n$  are diagonal then the  $n$ -tuple  $(Q_1, \dots, Q_n)$  is doubly stochastic if and only if the matrix  $A$  in (1.1.2) is doubly stochastic, that is, non-negative and has row and column sums 1.

In [2] Bapat conjectured what should be the mixed discriminant version of the van der Waerden inequality for permanents: if  $(Q_1, \dots, Q_n)$  is a doubly stochastic  $n$ -tuple then

$$D(Q_1, \dots, Q_n) \geq \frac{n!}{n^n} \tag{1.2.1}$$

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