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Concentration of the mixed discriminant of well-conditioned matrices

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ABSTRACT

We call an n-tuple $Q_1, \ldots, Q_n$ of positive definite $n \times n$ real matrices α-conditioned for some $\alpha \geq 1$ if for the corresponding quadratic forms $q_i : \mathbb{R}^n \longrightarrow \mathbb{R}$ we have $q_i(x) \leq \alpha q_i(y)$ for any two vectors $x, y \in \mathbb{R}^n$ of Euclidean unit length and $q_i(x) \leq \alpha q_j(x)$ for all $1 \leq i, j \leq n$ and all $x \in \mathbb{R}^n$. An n-tuple is called doubly stochastic if the sum of Q_i is the identity matrix and the trace of each Q_i is 1. We prove that for any fixed $\alpha \geq 1$ the mixed discriminant of an α-conditioned doubly stochastic n-tuple is $n^{O(1)} e^{-n}$. As a corollary, for any $\alpha \geq 1$ fixed in advance, we obtain a polynomial time algorithm approximating the mixed discriminant of an α-conditioned n-tuple within a polynomial in n factor.
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1. Introduction and main results

1.1. Mixed discriminants

Let $Q_1, \ldots, Q_n$ be $n \times n$ real symmetric matrices.

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The function $\det(t_1Q_1 + \ldots + t_nQ_n)$, where $t_1, \ldots, t_n$ are real variables, is a homogeneous polynomial of degree n in $t_1, \ldots, t_n$ and its coefficient

$$D(Q_1, \ldots, Q_n) = \frac{\partial^n}{\partial t_1 \cdots \partial t_n} \det(t_1Q_1 + \ldots + t_nQ_n) \tag{1.1.1}$$

is called the *mixed discriminant* of $Q_1, \ldots, Q_n$ (sometimes, the normalizing factor of $1/n!$ is used). Mixed discriminants were introduced by A.D. Alexandrov in his work on mixed volumes [1], see also [8]. They also have some interesting combinatorial applications, see Chapter V of [3].

Mixed discriminants generalize permanents. If the matrices $Q_1, \ldots, Q_n$ are diagonal, so that

$$Q_i = \operatorname{diag}(a_{i1}, \ldots, a_{in}) \quad \text{for} \quad i = 1, \ldots, n,$$

then

$$D(Q_1, \ldots, Q_n) = \operatorname{per} A \quad \text{where} \quad A = (a_{ij}) \tag{1.1.2}$$

and

$$\operatorname{per} A = \sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i\sigma(i)}$$

is the *permanent* of an $n \times n$ matrix A. Here the i-th row of A is the diagonal of Q_i and S_n is the symmetric group of all $n!$ permutations of the set $\{1, \ldots, n\}$.

1.2. Doubly stochastic n-tuples

If $Q_1, \ldots, Q_n$ are positive semidefinite matrices then $D(Q_1, \ldots, Q_n) \geq 0$, see [8]. We say that the n-tuple $(Q_1, \ldots, Q_n)$ is *doubly stochastic* if $Q_1, \ldots, Q_n$ are positive semidefinite,

$$Q_1 + \ldots + Q_n = I \quad \text{and} \quad \operatorname{tr} Q_1 = \ldots = \operatorname{tr} Q_n = 1,$$

where I is the $n \times n$ identity matrix and $\operatorname{tr} Q$ is the trace of Q. We note that if $Q_1, \ldots, Q_n$ are diagonal then the n-tuple $(Q_1, \ldots, Q_n)$ is doubly stochastic if and only if the matrix A in (1.1.2) is doubly stochastic, that is, non-negative and has row and column sums 1.

In [2] Bapat conjectured what should be the mixed discriminant version of the van der Waerden inequality for permanents: if $(Q_1, \ldots, Q_n)$ is a doubly stochastic n-tuple then

$$D(Q_1, \ldots, Q_n) \geq \frac{n!}{n^n} \tag{1.2.1}$$

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