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Robust global error bounds for uncertain linear inequality systems with applications

T.D. Chuong [*,1], V. Jeyakumar [2]

School of Mathematics and Statistics, University of New South Wales, Sydney, NSW 2052, Australia

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A B S T R A C T

In this paper, we examine global error bounds for a linear inequality system in the face of data uncertainty by building upon recent developments in robust optimization, where the uncertain parameters are assumed to be in prescribed uncertainty sets. We present necessary and sufficient dual conditions for the existence of *robust global error bounds*. As a consequence, we obtain a robust Hoffman error bound in the case of commonly used interval data uncertainty, extending the well-known Hoffman's error bound. We then introduce the notion of radius of robust global error bound under interval data uncertainty and present a formula for finding the radius by examining the stability of robust global error bounds. It provides the radius of the largest interval uncertainty set in which the uncertain system admits a robust global error bound. As an immediate application to robust linear programming, we provide conditions for robust feasibility and give a formula for radius of robust feasibility of an uncertain linear program.

* Corresponding author.
E-mail addresses: chuongthaidoan@yahoo.com (T.D. Chuong), v.jeyakumar@unsw.edu.au (V. Jeyakumar).

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1. Introduction

In this paper, we consider an *uncertain linear inequality* system

$$x \in \mathbb{R}^n, \quad a_j^\top x - b_j \leq 0, \quad j = 1, \ldots, p, \tag{SU}$$

where the data $a_j \in \mathbb{R}^n$ and $b_j \in \mathbb{R}$, $j = 1, \ldots, p$, are *uncertain* and they belong to the prescribed *uncertainty* sets U_j, $j = 1, \ldots, p$, which are assumed to be *compact* subsets in the space $\mathbb{R}^{n+1}$.

In the case where the uncertainty set U_j is a singleton, i.e. $U_j = \{(\bar{a}_j, \bar{b}_j\}$, for $j = 1, \ldots, p$, the system (SU) becomes an *uncertainty-free* linear inequality system,

$$x \in \mathbb{R}^n, \quad \bar{a}_j^\top x - \bar{b}_j \leq 0, \quad j = 1, \ldots, p, \tag{1.1}$$

which has been extensively studied in relation to global error bounds [1,16,20,21,23,25]. Here the data $\bar{a}_j \in \mathbb{R}^n$, $\bar{b}_j \in \mathbb{R}$, $j = 1, \ldots, p$, are fixed. In this case, the system (1.1) is said to admit a *global error bound* whenever there exists a real number $\tau > 0$ such that

$$d(x, S) \leq \tau[f(x)]_+ \text{ for all } x \in \mathbb{R}^n, \tag{1.2}$$

where $S = \{x \in \mathbb{R}^n \mid \bar{a}_j^\top x - \bar{b}_j \leq 0, \ j = 1, \ldots, p\}$ is the solution set of (1.1), $f(x) := \max\{\bar{a}_j^\top x - \bar{b}_j \mid j = 1, \ldots, p\}$, $[f(x)]_+ := \max\{f(x), 0\}$ and $d(\cdot, S)$ is the distance function from a variable point to the set S with the usual Euclidean norm. The infimum of all error bounds τ satisfying (1.2) is called the *best bound* or *error bound modulus* of (1.1).

It is well-known from Hoffman [15] that if $S \neq \emptyset$, then the system (1.1) has a global error bound; thus (1.2) holds for some $\tau > 0$. Various extensions of Hoffman's result have been given in the literature for convex inequality systems (see e.g., [26,28]) and also for more general inequality systems (see e.g., [8,19,22] and the references therein).

In reality, however, linear inequality systems often contain uncertain data. Data can be uncertain, for instance, due to measurement, prediction or estimation errors that come from the lack of knowledge of the parameters of the system. The mathematical approaches that deal with data uncertainty in inequality systems can be classified into two categories: (a) stochastic and (b) deterministic. Stochastic approaches describe uncertainty using probability distributions. The deterministic approach, that describes uncertainty via compact sets, has emerged as a powerful computationally tractable methodology, particularly, in the area of optimization, where it is known as Robust Optimization [2,3].

Following the deterministic approach [2,12,13,17,18], we say that the uncertain linear inequality system (SU) admits a *robust global error bound* (resp. *robust local error bound*) whenever its *robust* counterpart,

$$x \in \mathbb{R}^n, \quad a_j^\top x - b_j \leq 0, \quad \forall (a_j, b_j) \in U_j, \quad j = 1, \ldots, p, \tag{SR}$$