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Spectra of generalized corona of graphs

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Given simple graphs $G, H_1, \ldots, H_n$, where $n = |V(G)|$, the generalized corona, denoted $G\tilde{\circ} \overset{n}{\underset{i=1}{\Lambda}} H_i$, is the graph obtained by taking one copy of graphs $G, H_1, \ldots, H_n$ and joining the ith vertex of G to every vertex of H_i. In this paper, we determine and study the characteristic, Laplacian and signless Laplacian polynomial of $G\tilde{\circ} \overset{n}{\underset{i=1}{\Lambda}} H_i$. This leads us to construct new pairs of cospectral, L-cospectral and Q-cospectral graphs. As an application, we give a simple proof for Csikvari's Lemma on eigenvalues of graphs.

1. Introduction

Throughout the paper I_n and j_n are identity matrix of order n and length-n column vector consisting entirely of 1's, respectively. $\bar{K}_n$ stands for the graph with n isolated

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vertices and $diag(t_1, \ldots, t_n)$ is a diagonal matrix whose diagonal entries are $t_1, \ldots, t_n$. The notation $G = \emptyset$ means that the graph G has no vertices and no edges.

Let G be a graph with the vertex set $\{v_1, \ldots, v_n\}$. The adjacency matrix of G is an $(n \times n)$ matrix $A(G)$ whose (i, j)-entry is 1 if v_i is adjacent to v_j and 0, otherwise. The characteristic polynomial of G, denoted by $f_G(\lambda)$, is the characteristic polynomial of $A(G)$. We will write it simply f_G when there is no confusion. The roots of f_G are called the eigenvalues of G. The Laplacian matrix of G and the signless Laplacian matrix of G are defined as $L(G) = \Delta(G) - A(G)$ and $Q(G) = \Delta(G) + A(G)$, respectively, where $\Delta(G)$ is the diagonal matrix whose diagonal entries are degree sequences of G. We denote the Laplacian polynomial of G (L-polynomial) by $f_{L(G)}(\mu)$ and the signless Laplacian polynomial of G (Q-polynomial) by $f_{Q(G)}(\nu)$. Denote the eigenvalues of $A(G)$, $L(G)$ and $Q(G)$, respectively, by

$$\lambda_1(G) \geq \lambda_2(G) \geq \cdots \geq \lambda_n(G)$$
$$\mu_1(G) \leq \mu_2(G) \leq \cdots \leq \mu_n(G)$$
$$\nu_1(G) \leq \nu_2(G) \leq \cdots \leq \nu_n(G).$$

We recall that two graphs are cospectral (L-cospectral and Q-cospectral, respectively) if they have the same characteristic (Laplacian and signless Laplacian, respectively) polynomials.

Graph operations are natural techniques for producing new graphs from old ones, and their spectra have received considerable attention in recent years. The *corona* of G and H, denoted $G \circ H$, is the graph obtained by taking one copy of G and $|V(G)|$ copies of H, and joining the ith vertex of G to every vertex in the ith copy of H. This construction was first introduced by Frucht and Harary in [5] with the goal of constructing a graph whose automorphism group is the wreath product of the automorphism group of their components. Since then a number of papers on graph-theoretic properties of corona have appeared. As far as eigenvalues are concerned, the characteristic polynomial, L-polynomial and Q-polynomial of the corona of any two graphs can be expressed by that of two factor graphs [11,10,4,2]. Similarly, the characteristic polynomial, L-polynomial and Q-polynomial of edge corona, neighbourhood corona, subdivision-vertex and subdivision-edge neighbourhood corona of two graphs were completely computed in [6,9,8]. Let G and H be two graphs with n and m vertices, respectively. By a suitable labeling, Cam McLeman and Erin McNicholas in [10] obtained

$$f_{G \circ H}(\lambda) = (f_H(\lambda))^n f_G(\lambda - \chi_H(\lambda)),$$

where $\chi_H(\lambda)$, *coronal* of H, is the sum of all entries of the matrix $(\lambda I - A(H))^{-1}$ (or $\chi_H(\lambda) = j_m^T(\lambda I - A(H))^{-1} j_m$). Also Qun Liu in [7] obtained

$$f_{L(G \circ H)}(\mu) = (f_{L(H)}(\mu - 1))^n f_{L(G)}(\mu - m - \chi_{L(H)}(\mu)),$$