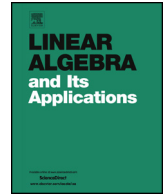




Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa



An S -type eigenvalue localization set for tensors



Chaoqian Li, Aiquan Jiao, Yaotang Li*

School of Mathematics and Statistics, Yunnan University, Yunnan, 650091, PR China

ARTICLE INFO

Article history:

Received 18 November 2015

Accepted 18 December 2015

Available online 29 December 2015

Submitted by R. Brualdi

MSC:

12E10

15A18

15A69

Keywords:

Tensor eigenvalue

Localization set

Positive definite

Positive semi-definite

ABSTRACT

An S -type eigenvalue localization set for a tensor is given by breaking $N = \{1, 2, \dots, n\}$ into disjoint subsets S and its complement. It is shown that the new set is tighter than those provided by Qi (2005) [21] and Li et al. (2014) [13]. As applications of the results, a checkable sufficient condition for the positive definiteness of tensors and a checkable sufficient condition of the positive semi-definiteness of tensors are given.

© 2015 Elsevier Inc. All rights reserved.

1. Introduction

Eigenvalue problems of tensors have become an important topic of study in numerical multilinear algebra, and they have a wide range of practical applications; see [2–5, 7–11, 18–20, 22, 23, 25, 26]. Here we call $\mathcal{A} = (a_{i_1 \dots i_m})$ a complex (real) tensor of order m dimension n , denoted by $\mathcal{A} \in C^{[m, n]}$ ($R^{[m, n]}$), if

* Corresponding author.

E-mail addresses: lichaoqian@ynu.edu.cn (C.Q. Li), jaq1029@163.com (A.Q. Jiao), liyaotang@ynu.edu.cn (Y.T. Li).

$$a_{i_1 \dots i_m} \in C(R),$$

where $i_j = 1, \dots, n$ for $j = 1, \dots, m$. Moreover, if there are a complex number λ and a nonzero complex vector $x = (x_1, x_2, \dots, x_n)^T$ such that

$$\mathcal{A}x^{m-1} = \lambda x^{[m-1]},$$

then λ is called an eigenvalue of $\mathcal{A}$ and x an eigenvector of $\mathcal{A}$ associated with λ [17,21], where $\mathcal{A}x^{m-1}$ is an n -dimensional vector whose i th component is

$$(\mathcal{A}x^{m-1})_i = \sum_{i_2, \dots, i_m \in N} a_{ii_2 \dots i_m} x_{i_2} \cdots x_{i_m} \quad (N = \{1, 2, \dots, n\})$$

and

$$x^{[m-1]} = (x_1^{m-1}, x_2^{m-1}, \dots, x_n^{m-1})^T.$$

If x and λ are all real, then λ is called an H-eigenvalue of $\mathcal{A}$ and x an H-eigenvector of $\mathcal{A}$ associated with λ [21,22,28].

One of many practical applications of eigenvalues of tensors is that one can identify the positive (semi-)definiteness for an even-order real symmetric tensor by using the smallest H-eigenvalue of a tensor, consequently, can identify the positive (semi-)definiteness of the multivariate homogeneous polynomial determined by this tensor, for details, see [11,21].

Because it is not easy to compute the smallest H-eigenvalue of tensors when the order and dimension are large, one always tries to give a set including all eigenvalues in the complex plane [13–16,21]. In particular, if this set for an even-order real symmetric tensor is in the right-half complex plane, then we can conclude that the smallest H-eigenvalue is positive, consequently, the corresponding tensor is positive definite.

In [21], Qi gave an eigenvalue localization set for real symmetric tensors, which is a generalization of the well-known Geršgorin's eigenvalue localization set of matrices [6,24]. This result can be easily generalized to general tensors [13,27].

Theorem 1. (See [13,21,27].) Let $\mathcal{A} = (a_{i_1 \dots i_m}) \in C^{[m,n]}$. Then

$$\sigma(\mathcal{A}) \subseteq \Gamma(\mathcal{A}) := \bigcup_{i \in N} \Gamma_i(\mathcal{A}),$$

where $\sigma(\mathcal{A})$ is the set of all the eigenvalues of $\mathcal{A}$,

$$\Gamma_i(\mathcal{A}) = \{z \in \mathbb{C} : |z - a_{i \dots i}| \leq r_i(\mathcal{A})\}, \quad r_i(\mathcal{A}) = \sum_{\substack{i_2, \dots, i_m \in N, \\ \delta_{ii_2 \dots i_m} = 0}} |a_{ii_2 \dots i_m}|$$

Download English Version:

<https://daneshyari.com/en/article/6416194>

Download Persian Version:

<https://daneshyari.com/article/6416194>

[Daneshyari.com](https://daneshyari.com)