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Eigenvectors of Hermitian Toeplitz matrices with smooth simple-loop symbols

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The paper is devoted to the structure and the asymptotics of the eigenvector matrix of Hermitian Toeplitz matrices with moderately smooth symbols which trace out a simple loop on the real line. The results extend existing results on banded Toeplitz matrices to full Toeplitz matrices with temperate decay of the entries in the first row and column. We establish formulas for both the exact and the asymptotic computation of arbitrarily prescribed individual components of arbitrarily prescribed individual eigenvectors. These formulas are in terms of the Wiener–Hopf factorization of a function which depends solely on the symbol and the corresponding eigenvalue or even only on an approximation to this eigenvalue. The paper does not aim at providing numerical algorithms. Its main purpose is rather to reveal the structure underneath the eigenvectors, which are shown to be the sum of two harmonics and certain edge corrections.

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1. Introduction

Eigenvalues and eigenvectors of large Toeplitz matrices are of interest in a large variety of applications, including statistical physics, quantum mechanics, and numerical analysis. In statistical physics, the matrix dimension n is beyond the dimensions one can tackle numerically with the help of computers, and when studying exactly solvable models of statistical physics, one is even forced to consider n as a variable which takes large but unspecified values. In numerical analysis, the n is in the hundreds or ten-thousands when actually performing computations, but when embarking on convergence analysis, one is again compelled to view n as a variable going to infinity.

In many problems of statistical physics one only needs certain averages or means of spectral data and not the individual eigenvalues or eigenvectors themselves. Things change in connection with quantum mechanics and numerical analysis. In quantum mechanics, one may ask whether certain states are extended or localized, which eventually is a question on the nature of certain individual eigenvectors. And in numerical analysis it is of course always beneficial and sometimes even indispensable to have information about the matrix V in the Jordan canonical form $A = VDV^{-1}$.

The study of averages of eigenvalues, or to state it in other words, of the collective behavior of the eigenvalues of Toeplitz matrices goes back to the celebrated limit theorems of Szegő. In addition, the extreme eigenvalues of Hermitian Toeplitz matrices have been thoroughly investigated by many authors, including Kac, Murdock, Szegő, Widom, Parter, Serra-Capizzano. It is only in the recent past that the entity of the individual eigenvalues of large Toeplitz matrices have attracted attention [1,2,4,7,10]. See also [6].

Individual eigenvectors of Toeplitz matrices were studied in [8,9,13,15–17]. Our present investigation is in the spirit of papers [8] and [15]. The former deals with certain banded Hermitian Toeplitz matrices, while the latter concerns a special class of full symmetric Toeplitz matrices, the so-called Kac–Murdock–Szegő matrices. We here describe the behavior of individual eigenvectors of sequences of increasing Toeplitz matrices under significantly weaker assumptions. In contrast to [8], we may now treat certain full Hermitian Toeplitz matrices with moderate decay of the entries in the first row and column, and in comparison with [15], we are able to tackle a much larger class of matrices.

The simplest Toeplitz matrices are the circulant matrices and the tridiagonal symmetric Toeplitz matrices. These are diagonalized by the Discrete Fourier Transform and by the Discrete Sine Transform of type I, respectively. See, e.g., [5]. In other words, the eigenvectors are very special complex or real harmonics. This will in general no longer be the case for the matrices considered here. Instead, the central contribution to the eigenvectors will be a linear combination of *two* harmonics.

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