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Modular adjacency algebras of Grassmann graphs

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ABSTRACT

The adjacency algebra of an association scheme is defined over an arbitrary field. In general, it is always semisimple over a field of characteristic zero but not always semisimple over a field of positive characteristic. The structures of adjacency algebras over fields of positive characteristic have not been sufficiently studied.

In this paper, we consider the structures of adjacency algebras of some P-polynomial schemes of class d with intersection numbers $c_i \not\equiv 0$ modulo p for $1 \leq i \leq d$ over fields of positive characteristic p. The classes of these P-polynomial schemes include association schemes originating from Grassmann graphs, double Grassmann graphs, and all types of dual polar graphs. We discuss the structures of the modular adjacency algebras of Grassmann graphs.

1. Introduction

An adjacency algebra of an association scheme is defined over an arbitrary field. In general, it is always semisimple over a field of characteristic zero but not always semisimple

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over a field of positive characteristic. An adjacency algebra of an association scheme over a field of positive characteristic is called a modular adjacency algebra. Hanaki and the second author of this paper determined the structure of the modular adjacency algebras and the modular standard modules of association schemes of class 2 [6]. Using modular standard modules, they provided more detailed classification than using parameters of strongly regular graphs. This indicates that structures of the modular standard modules of association schemes provide more detailed characterization than parameters of association schemes. In order to determine the structure of the modular standard modules, we first need to obtain the structure of the modular adjacency algebras. However, the structure of modular adjacency algebras has not been sufficiently studied (see [9,11,12]).

In this paper, we will consider the structure of modular adjacency algebras of P-polynomial schemes with the intersection numbers $c_i \not\equiv 0$ modulo p for $1 \leq i \leq d$. The class of these P-polynomial schemes includes association schemes originating from Grassmann graphs, double Grassmann graphs, and all types of dual polar graphs. These schemes have an additional condition for the intersection numbers: $b_i \equiv 0$ for $1 \leq i \leq d-1$. In particular, we will discuss the structure of the modular adjacency algebras of Grassmann graphs and determine the structure of the modular adjacency algebras of Grassmann graphs with some special parameters. Although Grassmann graphs are q-analogues of Johnson graphs, we obtain the correspondence of the structure of the modular adjacency algebras of Grassmann graphs to the structure of the modular adjacency algebras of Johnson graphs.

2. Preparation

2.1. Association schemes

Let X be a finite set with cardinality n. We define $R_0 := \{(x,x) \mid x \in X\}$. Let $R_i \subseteq X \times X$ be given. We set $R_{i^*} := \{(z,y) \mid (y,z) \in R_i\}$. Let S be a partition of $X \times X$ such that $R_0 \in S$ and the empty set $\emptyset \notin S$, and we assume that $R_{i^*} \in S$ for each $R_i \in S$. Then, the pair $\mathfrak{X} = (X, S)$ will be called an association scheme [1] if, for all $R_i, R_j, R_k \in S$, there exists a cardinal number p_{ij}^k such that, for all $(y,z) \in R_k$,

$$\sharp\{x \in X \bigm| (y,x) \in R_i, (x,z) \in R_j\} = p_{ij}^k.$$

The numbers p_{ij}^k are called the intersection numbers of $\mathfrak{X}$. The valency of R_i is the intersection number $p_{ii^*}^0$, and it is denoted by n_i.

For each $R_i \in S$, we define the $n \times n$ matrix A_i indexed by the elements of X, as

$$(A_i)_{xy} = \begin{cases} 1 & \text{if } (x,y) \in R_i, \\ 0 & \text{otherwise.} \end{cases}$$

We call A_i the adjacency matrix of R_i.