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On the unitary Cayley graphs of matrix algebras

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A B S T R A C T

We study a family of Cayley graphs on the group of $n \times n$ matrices $M_n(F)$, where F is a finite field and n is a natural number, with the connection set of $GL_n(F)$. We find that this graph is strongly regular only when $n = 2$. We find diameter of this graph and we show that, every matrix in $M_n(F)$ is either invertible or sum of two invertible matrices. Moreover, we show that $G_{M_n(F)}$ is class 1 if and only if $char\, F = 2$. Finally, it is shown that for each graph G and each finite field F, G is an induced subgraph of $Cay(M_n(F), GL_n(F))$.

1. Introduction

Let F be a finite field, and n be a natural number. Let $M_n(F)$ and $GL_n(F)$ denote the set of $n \times n$ matrices over F and the set of $n \times n$ invertible matrices over F, respectively.

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The *unitary Cayley graph* of additive group $M_n(F)$, $G_{M_n(F)} = Cay(M_n(F), GL_n(F))$, is the Cayley graph whose vertex set is $M_n(F)$ and edge set is $\{\{A,B\} : A,B \in M_n(F) \text{ and } det(A-B) \neq 0\}$. The chromatic number, clique number and independence number of $G_{M_n(F)}$ are given in [6] along with other results. For some other recent papers on unitary Cayley graphs, we refer the reader to [1,5–7].

Theorem 1.1. *(See [6].) Let F be a finite field, and n be a positive integer. Then $\omega(G_{M_n(F)}) = |F|^n$.*

Theorem 1.2. *(See [6].) Let F be a finite field, and n be a positive integer. Then $\alpha(G_{M_n(F)}) = |F|^{n^2-n}$.*

An *edge regular graph* (erg) with parameters (n,k,λ) is a graph with n vertices that is regular of valency k and that has the following property:

- For any two adjacent vertices x, y, there are exactly λ vertices adjacent to both x and y.

A graph of order n is called *strongly regular graph* (srg) with parameters (n,k,λ,μ) whenever

- Each vertex is adjacent to k vertices.
- For any two adjacent vertices x, y, there are exactly λ vertices adjacent to both x and y.
- For any two non-adjacent vertices x, y, there are exactly μ vertices adjacent to both x and y.

Let F be a finite field of order q. A matrix in $M_n(F)$ is a linear derangement if it is invertible and does not fix any non-zero vector. Such a matrix is characterized as not having 0 or 1 as an eigenvalue. Let e_n be the number of linear derangements and define $e_0 = 1$. Recall from [9] that e_n satisfies the recursion

$$e_n = e_{n-1}\left(q^n - 1\right)q^{n-1} + (-1)^n q^{n(n-1)/2}. \tag{1}$$

2. Unitary Cayley graph over $M_n(F)$

Lemma 2.1. *Let n be a natural number, and F be a finite field. The unitary Cayley graph $G_{M_n(F)}$ is $|GL_n(F)|$-regular and every two adjacent vertices of $G_{M_n(F)}$ has e_n common neighbors.*

Proof. Since $G_{M_n(F)} = Cay(M_n(F), GL_n(F))$ is a Cayley graph, it is well known that $G_{M_n(F)}$ is $|GL_n(F)|$-regular. Assume that A and B are adjacent. The set of common neighbors of A and B is $N(A) \cap N(B) = (A + GL_n(F)) \cap (B + GL_n(F))$. Let $\varphi : (A + GL_n(F)) \cap (B + GL_n(F)) \longrightarrow ((A-B) + GL_n(F)) \cap GL_n(F)$ be defined