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Matrix polynomials with specified eigenvalues

Michael Karow[a], Emre Mengi[b,*]

[a] Department of Mathematics, TU-Berlin, Sekretariat MA 4-5, Strasse des 17. Juni 136, D-10623, Berlin, Germany

[b] Department of Mathematics, Koç University, Rumelifeneri Yolu, 34450 Sarıyer-İstanbul, Turkey

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ABSTRACT

This work concerns the distance in the 2-norm from a given matrix polynomial to a nearest polynomial with a specified number of its eigenvalues at specified locations in the complex plane. Initially, we consider perturbations of the constant coefficient matrix only. A singular value optimization characterization is derived for the associated distance. We also consider the distance in the general setting, when all of the coefficient matrices are perturbed. In this general setting, we obtain a lower bound in terms of another singular value optimization problem. The singular value optimization problems derived facilitate the numerical computation of the distances.

* Corresponding author.
E-mail addresses: karow@math.tu-berlin.de (M. Karow), emengi@ku.edu.tr (E. Mengi).

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1. Introduction

We study the distance from a given matrix polynomial to a nearest matrix polynomial with a specified number of its eigenvalues at specified locations in the complex plane. Formally, suppose $P : \mathbb{C} \to \mathbb{C}^{n\times n}$ denotes a given polynomial defined by

$$P(\lambda) := \sum_{j=0}^{m} \lambda^j A_j. \tag{1}$$

Here $A_j \in \mathbb{C}^{n\times n}$, $j = 1, \ldots, m$ are fixed, and we assume A_m is full rank. Furthermore, suppose that a set $\mathbb{S} := \{\lambda_1, \ldots, \lambda_s\}$ consisting of complex scalars, and a positive integer r, are given. We provide a singular value optimization characterization for the distance

$$\tau_r(\mathbb{S}) := \inf\left\{ \|\Delta\|_2 \;\middle|\; \Delta \in \mathbb{C}^{n\times n} \text{ s.t. } \sum_{j=1}^{s} m_j(P+\Delta) \geq r \right\}. \tag{2}$$

Above $m_j(P + \Delta)$ denotes the algebraic multiplicity of λ_j as an eigenvalue of $\widetilde{P}(\lambda) := P(\lambda) + \Delta$, i.e., the multiplicity of λ_j as a root of the characteristic polynomial $\det(\widetilde{P}(\lambda))$. We also consider this distance in the general setting, when perturbations of every coefficient A_j, $j = 0, \ldots, m$, of $P(\lambda)$ are admissible. Another singular value optimization characterization yielding a lower bound for the distance in this general setting is derived.

The characterizations derived here are generalizations of the singular value optimization characterization for a linear matrix pencil of the form $L(\lambda) = A_0 + \lambda A_1$ in [16], which was inspired by [19]. Unlike that in [19], the derivation here fully depends on a Sylvester equation characterization for the matrix polynomial P to have sufficiently many eigenvalues belonging to $\mathbb{S}$. This yields a neater derivation. The machinery here and in [16] have similarities, but considerably more work is required here. Some of the additional machinery that we depend on in this matrix polynomial setting are as follows:

(1) The Sylvester equation characterizations are derived starting from the characterizations in [16], but the derivation employs the companion form linearizations of matrix polynomials in Section 2;

(2) The singular value optimization characterizations originate from a subtle relation between a pair of optimal left and right singular vectors. The details are worked out in Section 3.2. Analogous relations are observed in [19,16] for linear pencils, but the procedure for the same observation for an arbitrary matrix polynomial is more involved;

(3) The divided difference formulas in Section 3.5 provide means to express the derived singular value characterization in a comprehensible fashion. They are not needed in the linear pencil setting.