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Solvable Leibniz algebras with triangular nilradicals

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ABSTRACT

In this paper the description of solvable Lie algebras with triangular nilradicals is extended to Leibniz algebras. It is proven that the matrices of the left and the right operators on the elements of Leibniz algebra have the upper triangular forms. We establish that solvable Leibniz algebra of a maximal possible dimension with a given triangular nilradical is a Lie algebra. Furthermore, solvable Leibniz algebras with triangular nilradicals of the low dimensions are classified.

1. Introduction

Leibniz algebras were introduced at the beginning of the 90s of the past century by J.-L. Loday [3]. They are a generalization of well-known Lie algebras, which admit a remarkable property that an operator of the right multiplication is a derivation.

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From the classical theory of Lie algebras it is well known that the study of finite-dimensional Lie algebras was reduced to the nilpotent ones [11,12]. In the Leibniz algebra case there is an analogue of Levi's theorem [4]. Namely, the decomposition of a Leibniz algebra into a semidirect sum of its solvable radical and a semisimple Lie algebra is obtained. The semisimple part can be described from simple Lie ideals (see [5]) and therefore, the main focus is to study the solvable radical.

The analysis of several works devoted to the study of solvable Lie algebras (for example [1,2,13–15], where solvable Lie algebras with various types of the nilradical were studied, such as naturally graded filiform and quasi-filiform algebras, Abelian, triangular, etc.) shows that we can also apply similar methods to solvable Leibniz algebras with a given nilradical. In fact, any solvable Lie algebra can be represented as an algebraic sum of a nilradical and its complimentary vector space. Mubarakzjanov proposed a method, which claims that the dimension of the complimentary vector space does not exceed the number of nil-independent derivations of the nilradical [12]. Extension of this method to Leibniz algebras is shown in [6]. Usage of this method yields a classification of solvable Leibniz algebras with the given nilradicals in [6–10].

In this article we present the description of solvable Leibniz algebras whose nilradical is a Lie algebra of upper triangular matrices. Since in the work [14] solvable Lie algebras with the triangular nilradical are studied, we reduce our study to non-Lie Leibniz algebras.

Recall that in [14] solvable Lie algebras with the triangular nil-radicals of minimum and maximum possible dimensions were described. Moreover, uniqueness of a Lie algebra of maximal possible dimension with the given triangular nilradical is established.

In order to realize a goal of our study we organize the paper as follows. In Section 2 we give the necessary preliminary results. Section 3 is devoted to the description of finite-dimensional solvable Leibniz algebras with the upper triangular nilradical. We establish that such Leibniz algebras of minimum and maximum possible dimensions are Lie algebras. Finally, in Section 4 we present the complete description of the results of Section 3 in the low dimensions.

Throughout the paper we consider finite-dimensional vector spaces and algebras over the field $\mathbb{C}$. Moreover, in a multiplication table of an algebra omitted products are assumed to be zero and if it is not stated otherwise, we will consider non-nilpotent solvable algebras.

2. Preliminaries

In this section we give the basic concepts and the results used in the studying of Leibniz algebras with the triangular nilradicals.

Definition 2.1. An algebra $(L, [-,-])$ over a field F is called a Leibniz algebra if for any $x, y, z \in L$ the so-called Leibniz identity