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Cospectrality of graphs

Alireza Abdollahi [a,b,*], Mohammad Reza Oboudi [a,b]

[a] *Department of Mathematics, University of Isfahan, Isfahan 81746-73441, Iran*
[b] *School of Mathematics, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5746, Tehran, Iran*

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A B S T R A C T

Richard Brualdi proposed in Stevanivić (2007) [6] the following problem:
(Problem AWGS.4) Let G_n and G'_n be two nonisomorphic graphs on n vertices with spectra

$$\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_n \quad \text{and} \quad \lambda'_1 \geqslant \lambda'_2 \geqslant \cdots \geqslant \lambda'_n,$$

respectively. Define the distance between the spectra of G_n and G'_n as

$$\lambda(G_n, G'_n) = \sum_{i=1}^{n} (\lambda_i - \lambda'_i)^2 \quad \left(\text{or use } \sum_{i=1}^{n} |\lambda_i - \lambda'_i|\right).$$

Define the cospectrality of G_n by

$$\text{cs}(G_n) = \min\{\lambda(G_n, G'_n)\colon\ G'_n \text{ not isomorphic to } G_n\}.$$

Let

$$\text{cs}_n = \max\{\text{cs}(G_n)\colon\ G_n \text{ a graph on } n \text{ vertices}\}.$$

Problem A. Investigate $\text{cs}(G_n)$ for special classes of graphs.

Problem B. Find a good upper bound on cs_n.

In this paper we study Problem A and determine the cospectrality of certain graphs by the Euclidian distance.

* Corresponding author.
E-mail addresses: a.abdollahi@math.ui.ac.ir (A. Abdollahi), mr.oboudi@sci.ui.ac.ir (M.R. Oboudi).

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Let K_n denote the complete graph on n vertices, nK_1 denote the null graph on n vertices and $K_2 + (n-2)K_1$ denote the disjoint union of the K_2 with $n-2$ isolated vertices, where $n \geqslant 2$. In this paper we find $\mathrm{cs}(K_n)$, $\mathrm{cs}(nK_1)$, $\mathrm{cs}(K_2 + (n-2)K_1)$ $(n \geqslant 2)$ and $\mathrm{cs}(K_{n,n})$.

1. Introduction

Throughout the paper all graphs are simple, that is finite and undirected without loops and multiple edges. By the spectrum of a graph G, we mean the multiset of eigenvalues of adjacency matrix of G.

Richard Brualdi proposed in [6] the following problem:

(Problem AWGS.4) Let G_n and G'_n be two nonisomorphic graphs on n vertices with spectra

$$\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_n \quad \text{and} \quad \lambda'_1 \geqslant \lambda'_2 \geqslant \cdots \geqslant \lambda'_n,$$

respectively. Define the distance between the spectra of G_n and G'_n as

$$\lambda(G_n, G'_n) = \sum_{i=1}^{n} (\lambda_i - \lambda'_i)^2 \quad \left(\text{or use } \sum_{i=1}^{n} |\lambda_i - \lambda'_i|\right).$$

Define the cospectrality of G_n by

$$\mathrm{cs}(G_n) = \min\{\lambda(G_n, G'_n):\ G'_n \text{ not isomorphic to } G_n\}.$$

Thus $\mathrm{cs}(G_n) = 0$ if and only if G_n has a cospectral mate. Let

$$\mathrm{cs}_n = \max\{\mathrm{cs}(G_n):\ G_n \text{ a graph on } n \text{ vertices}\}.$$

This function measures how far apart the spectrum of a graph with n vertices can be from the spectrum of any other graph with n vertices.

Problem A. Investigate $\mathrm{cs}(G_n)$ for special classes of graphs.

Problem B. Find a good upper bound on cs_n.

In this paper we study Problem A and determine the cospectrality of certain graphs by the Euclidian distance, that is

$$\lambda(G_n, G'_n) = \sum_{i=1}^{n} (\lambda_i - \lambda'_i)^2.$$