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Sharp coincidences for absolutely summing multilinear operators

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ABSTRACT

In this paper we prove the optimality of a family of known coincidence theorems for absolutely summing multilinear operators. We connect our results with the theory of multiple summing multilinear operators and prove the sharpness of similar results obtained via the complex interpolation method.

1. Preliminaries and background

A long standing problem raised by S. Banach [2, p. 40] asks whether in every infinite-dimensional Banach space there exists an unconditionally convergent series which fails to be absolutely convergent. The positive solution, in 1950, due to A. Dvoretzky and C.A. Rogers [15], is probably the main motivation for the appearance of the concept of absolutely summing operators in the 1950–1960s with the works of A. Grothendieck [17], A. Pietsch [31] and J. Lindenstrauss and A. Pełczyński [21].

Essentially, if E and F are Banach spaces, an absolutely summing operator $u : E \to F$ is a linear operator that improves the convergence of series in the following fashion: each unconditionally summable sequence $(x_n)_{n=1}^{\infty}$ in E is sent to an absolutely summable sequence $(u(x_n))_{n=1}^{\infty}$ in F. More generally, if $1 \leqslant p \leqslant q < \infty$, a continuous linear operator $u : E \to F$ is absolutely $(q; p)$-summing if $(u(x_j))_{j=1}^{\infty} \in \ell_q(F)$ whenever

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$$\sup_{\varphi \in B_{E^*}} \sum_{j=1}^{\infty} \left|\varphi(x_j)\right|^p < \infty,$$

where E^* denotes the topological dual of E and B_{E^*} represents its closed unit ball (for the theory of absolutely summing operators we refer to [12] and, for recent results, [9,13] and references therein).

The space of all absolutely $(q;p)$-summing operators from E to F is denoted by $\Pi_{q;p}(E;F)$ (or $\Pi_p(E;F)$ if $p=q$). It is not difficult to prove that if $1 \leqslant p \leqslant q < \infty$, then $\Pi_p \subset \Pi_q$. Henceforth the space of all bounded linear operators from a Banach space E to a Banach space F will be represented by $\mathcal{L}(E;F)$.

The theory of absolutely summing operators is nowadays a mandatory topic in modern Banach Space Theory, with somewhat unexpected applications. For example, using tools of the theory of absolutely summing operators we can prove that if $E=\ell_1$ or $E=c_0$ every normalized unconditional basis is equivalent to the unit vector basis of E (see [21]). According to Pietsch [33, p. 365], one of the most profound results in Banach Space Theory is Grothendieck's *theorème fondamental de la théorie métrique des produits tensoriels*, from the famous Grothendieck's Résumé [17] (see also [11] for a modern approach), which can be rewritten in the language of absolutely summing operators as follows:

Theorem 1.1 *(Grothendieck). Every bounded linear operator from ℓ_1 to any Hilbert space is absolutely summing.*

This kind of result, in the modern terminology, is called *coincidence result*. The notion of cotype of a Banach space appeared in the 70s with works of J. Hoffmann-Jørgensen [18], B. Maurey [24], S. Kwapień [20], E. Dubinsky, A. Pełczyński, H.P. Rosenthal [14], among others. A Banach space E has cotype $s \in [2,\infty)$ if there is a constant $C \geqslant 0$ so that, for all positive integer n and all $x_1,\dots,x_n$ in E, we have

$$\left(\sum_{i=1}^{n} \|x_i\|^s\right)^{1/s} \leqslant C\left(\int_0^1 \left\|\sum_{i=1}^{n} r_i(t)x_i\right\|^2 dt\right)^{1/2}, \tag{1.1}$$

where, for all i, r_i represents the i-th Rademacher function. By $\cot E$ we denote the infimum of the cotypes assumed by E, i.e.,

$$\cot E := \inf\{2 \leqslant q < \infty;\ E \text{ has cotype } q\}.$$

It is important to recall that the infimum in the definition of $\cot E$ may not be achieved by E. The straight relation between cotype and absolutely summing operators can be seen in numerous works. For instance:

- (B. Maurey and G. Pisier [25]) If $\dim E = \infty$, then
$$\cot E = \inf\bigl\{r:\ \Pi_{r;1}(E;E) = \mathcal{L}(E;E)\bigr\}.$$
- (M. Talagrand [36,37]) A Banach space has cotype $s>2$ if and only if
$$\Pi_{s;1}(E;E) = \mathcal{L}(E;E)$$
and the result fails for $s=2$.
- (B. Maurey [24], L. Schwartz [35]) If F is an infinite-dimensional Banach space with cotype $s>2$, then
$$\inf\bigl\{r:\ \Pi_r\bigl(C(K);F\bigr) = \mathcal{L}\bigl(C(K);F\bigr)\bigr\} = s \tag{1.2}$$
and the infimum is *not* attained.
- (G. Botelho et al. [9]) If $2 \leqslant r < \cot F$ and $\dim E = \dim F = \infty$, then
$$\mathcal{L}(E,F) = \Pi_{q,r}(E,F) \quad \Rightarrow \quad \mathcal{L}(\ell_1,\ell_{\cot F}) = \Pi_{q,r}(\ell_1,\ell_{\cot F}). \tag{1.3}$$