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On the K-theory of feedback actions on linear systems ☆

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ABSTRACT

A categorical approach to linear control systems is introduced. Feedback actions on linear control systems arise as symmetric monodical category S_R. Stable feedback isomorphisms generalize dynamic enlargement of pairs of matrices. Subcategory of locally Brunovsky linear systems B_R is studied. We prove that the stable feedback isomorphisms of locally Brunovsky linear systems are characterized by the Grothendieck group $K_0(B_R)$. Hence a link from linear dynamical systems to algebraic K-theory is established.

1. Introduction

This paper deals with the study of feedback actions on a linear control system. Concrete description of the feedback classification of constant linear systems by means of sets of invariants and canonical forms goes back to the seminal works by Kalman, Casti and Brunovsky (see the fundamental references [5,7,11]), but the more general framework of parametrized families of linear systems [8,14] is proved to be a hard task in [4]. We restrict ourselves to the class of locally Brunovsky systems because a complete description of feedback invariants is available (see [6]).

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On the other hand we also are interested in the so-called dynamic feedback equivalence (see [3,9,10] as main references). This dynamic study is based on the addition of some suitable ancillary variables to systems [2, Ch. 4]. We introduce the notion of stable feedback equivalence and show that it is a generalization of both feedback and dynamic feedback equivalence. This generalization does not go too far, because if the base ring is a field, then feedback, dynamic feedback and stable feedback equivalence are the same notion.

We think an adequate tool to study the above subjects is category theory. First of all, the definition of the category S_R of linear systems over a commutative ring and feedback actions arises in a natural way, moreover feedback equivalences are the isomorphisms in the category. Dynamic enlargements and stabilization of linear systems are consequence of some bi-product (both product and coproduct) in the category and hence a symmetric monoidal structure in S_R is obtained. Finally the stable equivalences are the stable isomorphisms in the category. Consequently the invariants characterizing the stable equivalence will be collected in the K_0 group of the K-theory of the category, which is just the Grothendieck group completion of the monoidal structure when possible (i.e. when the isomorphism classes in the category are sets).

The paper is organized as follows. Section 2 is devoted to review main definitions used in the paper: linear system, feedback isomorphism, direct sum of linear systems and dynamic isomorphism. These notions generalize respectively the standard notions of pair of matrices, feedback equivalence, dynamic enlargement and dynamic feedback equivalence. We also define stable isomorphism of linear systems as adequate generalization of both feedback and dynamic isomorphism.

Section 3 is the core section of the paper devoted to the stable classification of linear systems. We prove that the pair

$$S_R = (\text{linear systems, feedback morphisms})$$

is a category whose isomorphisms are precisely feedback isomorphisms defined in [6]. Thus feedback classification of linear systems is just given by the classes of isomorphisms $(S_R)^{\text{iso}}$. Reachable systems A_R and locally Brunovsky systems B_R arise as subcategories of S_R equipped with the same homomorphisms: the feedback actions.

We define the operation $\oplus$ on linear systems and show that: (a) 'sum' operation $\oplus$ is both the categorical product and coproduct in the category of linear systems; (b) dynamic enlargement of a linear system now arises as the 'sum' of the system with a trivial one; and (c) category of linear systems equipped with $\oplus$ operation are symmetric monoidal, see [12] or [15].

Section 3 concludes with a characterization of stable equivalence in B_R (locally Brunovsky systems) in terms of first K-theory group $K_0(B_R)$ of the category.

Finally, Section 4 is devoted to compute effectively $K_0(B_R)$ as the Grothendieck group completion of the monoid $(B_R)^{\text{iso}}$ of locally Brunovsky systems up to feedback isomorphisms.

2. Stable feedback isomorphisms between linear systems

Let R be a commutative ring with $1 \neq 0$. In this section we introduce the dynamic and stable feedback isomorphisms of linear systems over R.

Definition 2.1. (See [8].) A linear system is a triple $\Sigma = (X, f, B)$ where X is an R-module, $f : X \to X$ an endomorphism and $B \subset X$ a submodule.

Definition 2.2. (See [6].) Two linear systems $\Sigma_1 = (X_1, f_1, B_2)$ and $\Sigma_2 = (X_2, f_2, B_2)$ are feedback isomorphic (f.i.) if there exists an isomorphism of R-modules between the space states $\phi : X_1 \to X_2$ such that

1. $\phi(B_1) = B_2$,
2. $Im(f_2 \circ \phi - \phi \circ f_1) \subset B_2$.

Remark 2.3. Recall that all pairs of matrices (A, B) define a linear system

$$(A, B) \mapsto \Sigma_{A,B} = \left(R^n, A, Im(B)\right) \tag{1}$$