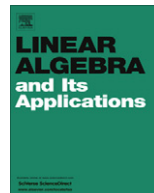




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More on G-matrices

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ABSTRACT

The first author and Hall defined recently a G-matrix as a real nonsingular matrix A such that there exist diagonal matrices D_1 and D_2 for which $(A^T)^{-1} = D_1 A D_2$. The class of G-matrices was shown to possess interesting properties. In this paper, some new characterizations are found and extensions to rectangular and complex matrices are discussed.

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1. Introduction

The first author and Hall defined in [2] the class of so called G-matrices. There, a real nonsingular matrix A was called G-matrix if there exist diagonal matrices D_1 and D_2 such that $(A^T)^{-1} = D_1 A D_2$.

It was observed in [2] that the following holds:

Theorem 1.1. *All orthogonal matrices are G-matrices.*

Theorem 1.2. *All nonsingular diagonal matrices are G-matrices.*

Theorem 1.3. *If A is a G-matrix, then both A^T and A^{-1} are G-matrices.*

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Theorem 1.4. *If A is an $n \times n$ G-matrix and D is an $n \times n$ nonsingular diagonal matrix, then both AD and DA are G-matrices.*

Theorem 1.5. *If A is an $n \times n$ G-matrix and P is an $n \times n$ permutation matrix, then both AP and PA are G-matrices.*

Theorem 1.6. *If A is a G-matrix, then both matrices A and A^{-T} have the same zero, non-zero structure. Thus the zero, non-zero structures of A and A^{-1} are symmetric to each other.*

Theorem 1.7. *The direct sum of G-matrices is again a G-matrix.*

The product form of the definition leads to the following two theorems.

Theorem 1.8. *Compound matrices of a G-matrix are also G-matrices.*

Theorem 1.9. *The Kronecker product of G-matrices is a G-matrix.*

Also, the following theorem was proved.

Theorem 1.10. *A 2×2 matrix is G-matrix if and only if it is nonsingular and has four or two nonzero entries.*

As is well known, Cauchy matrices are matrices of the form $C = [c_{ij}]$, where $c_{ij} = \frac{1}{x_i + y_j}$ for some numbers x_i and y_j . We shall restrict ourselves to square, say $n \times n$, Cauchy matrices. Of course, such matrices are defined only if $x_i + y_j \neq 0$ for all pairs of indices i, j , and it is well known that C is nonsingular if and only if all the numbers x_i are mutually distinct and all the numbers y_j are mutually distinct. In [2], the following theorem was proved.

Theorem 1.11. *Every nonsingular Cauchy matrix C for which both vectors $C^{-1}e$ and $C^{-T}e$ have no coordinate zero is a G-matrix; here, e is the vector of all ones.*

Let us recall [1, Observation 1] that the assumptions about the vectors $C^{-1}e$ and $C^{-T}e$ are superfluous. Indeed, it is well known that if $C = \frac{1}{x_i + y_j}$, then the inverse $C^{-1} = [\gamma_{ij}]$, where

$$\gamma_{ij} = (x_j + y_i) \frac{\prod_{\ell \neq i} (x_j + y_\ell) \prod_{k \neq j} (y_i + x_k)}{\prod_{\ell \neq j} (x_j - x_\ell) \prod_{k \neq i} (y_i - y_k)}. \tag{1}$$

The right-hand side of (1) can be written as $\frac{1}{x_j + y_i} U_j V_i$, where

$$U_j = (x_j + y_j) \prod_{k \neq j} \frac{x_j + y_k}{x_j - x_k}, \tag{2}$$

$$V_i = (x_i + y_i) \prod_{k \neq i} \frac{y_i + x_k}{y_i - y_k}. \tag{3}$$

If we thus define the diagonal matrices D_1, D_2 as $D_1 = \text{diag}(U_1, \dots, U_n)$, $D_2 = \text{diag}(V_1, \dots, V_n)$, then indeed $(C^T)^{-1} = D_1 C D_2$. Thus C is a G-matrix.

2. Results

Let us start with the following observation.

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