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Combining a hybrid preconditioner and a optimal adjustment algorithm to accelerate the convergence of interior point methods

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ABSTRACT

In this work, the optimal adjustment algorithm for p coordinates, which arose from a generalization of the optimal pair adjustment algorithm is used to accelerate the convergence of interior point methods using a hybrid iterative approach for solving the linear systems of the interior point method. Its main advantages are simplicity and fast initial convergence. At each interior point iteration, the preconditioned conjugate gradient method is used in order to solve the normal equation system. The controlled Cholesky factorization is adopted as the preconditioner in the first outer iterations and the splitting preconditioner is adopted in the final outer iterations. The optimal adjustment algorithm is applied in the preconditioner transition in order to improve both speed and robustness. Numerical experiments on a set of linear programming problems showed that this approach reduces the total number of interior point iterations and running time for some classes of problems. Furthermore, some problems were solved only when the optimal adjustment algorithm for p coordinates was used in the change of preconditioners.

1. Introduction

Interior point methods have been the object of intensive research since the appearance of their first polynomial version in the 80's. Their good practical performance and theoretical properties have

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motivated the implementation of sophisticated codes to solve large scale linear programming problems. The success of these methods rely mainly on the fact that the convergence is achieved in relatively few iterations [1].

Each iteration of an interior point method involves the solution of one or more linear system [2–4]. This is the most expensive step of these methods. There are several approaches for solving the linear systems. Usually, direct methods are adopted by reducing the indefinite augmented system to the normal equation one and applying the Cholesky factorization [5,6,2,3]. However, this approach cannot be applied for some classes of large scale problems due to memory and/or time limitations. For these problems, the iterative method for the solution of the linear system would be the chosen approach [7–10].

In this work, an iterative hybrid approach is used to solve the normal equation system that arises in an interior point method for linear programming. The conjugate gradient method is preconditioned during the initial interior point iterations using a kind of incomplete factorization called controlled Cholesky factorization [11], and in the remaining iterations, when these systems become highly ill-conditioned, using a specially tailored preconditioner, the splitting preconditioner developed in [10].

The transition between both preconditioners is critical in the sense that if it happens too early, the splitting preconditioner is not yet fit for the job. However, if it happens too late, the controlled Cholesky factorization preconditioner is no longer effective [12]. In this situation, the method would fail.

On the other hand, a new trend in the past few years consists in the use of simple linear programming methods in order to give a warm starting point for interior point methods, reducing the total number of iterations [13]. The von Neumann's algorithm is one of the first used in such applications since its iteration is very cheap and it has fast initial convergence.

In order to deal with this problem, we perform a few iterations of the optimal adjustment algorithm for p coordinates, between some interior point method iterations in order to improve the solution. In particular, it is applied just before the change of preconditioners, to improve the current point and to deliver a point closer to an optimal solution for the splitting preconditioner. This approach close the gap in the transition of preconditioners for some tested problems.

The optimal adjustment algorithm for p coordinates was proposed in [14] and it is a generalization of the optimal pair adjustment algorithm developed in [13], which is based on the classical von Neumann's algorithm. It has interesting properties such as simplicity and fast initial convergence, which motivated its use.

Numerical experiments with large scale linear programming problems show that such strategy allows to improve performance, achieving both faster convergence and adding robustness to the whole approach.

The paper is organized as follows. In Section 2, the primal–dual interior point methods are briefly reviewed and the linear systems that need to be solved are presented. In Section 3, the hybrid preconditioner, controlled Cholesky factorization and splitting preconditioner are presented and some features of these preconditioners are discussed. In Section 4, simple algorithms for linear programming such as the von Neumann's algorithm, the optimal pair adjustment algorithm and the optimal adjustment algorithm for p coordinates are described and several theoretical properties are discussed. Section 5 describes the computational experiments. In Section 6, conclusions are drawn and future perspectives are suggested.

2. Primal–dual interior-point methods

Consider the linear programming problem in the standard form,

$$\begin{aligned} \text{Min } & c^T x \\ \text{s.t. } & Ax = b \\ & x \geqslant 0, \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, $x \in \mathbb{R}^n$ and $m \leqslant n$. The dual problem associated with problem (1) is